Seismic Stability Analysis for Multi-Story Steel Frame with Inelastic Sidesway

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ABSTRACT

The critical load of a steel column in a multistory frame in elastic stage can be easily determined by elastic stability analysis. However, the critical load and effective length factor *K* of the column in a frame in inelastic stage cannot be evaluated directly with an alignment chart or any other methods based on elastic behavior. In this study, inelastic stability characteristic equations are derived for thirteen modes of a steel column on a frame associated with plastic hinges and failure hinges on a column or beam end. The critical load and effective length factor *K* of a column on a frame in inelastic stage is determined by solving the inelastic stability characteristic equations with numerical process. The results show that, in some modes, the value of a steel column's inelastic effective length factor *K* of is larger than the value of its effective length factor *K* in an elastic stage. Therefore, in some modes, the value of steel column's inelastic critical buckling load is less than its elastic critical buckling load. As a result, analyzing only the elastic stability of a frame's steel columns while disregarding their inelastic stability could lead one to overestimate the steel frame's seismic lateral resistant capacity.

INTRODUCTION

The elastic effective length *KL* of a steel column can be determined by elastic stability analysis. For the sake of convenient, an alignment chart of effective length factor *K* was developed in 1970s [1]. However, the alignment chart is only applicable if it is assumed that the beams and columns of a frame remain in an elastic stage, that is, if no plastic hinge is allowed to occur at the end of members. However, during a severe earthquake, strong ground motion may generate bending moment that produce plastic hinges at the ends of beams or columns. Under these conditions, the effective length factor *K* of steel columns cannot be evaluated by an alignment chart or any other method based on the assumptions of elastic behavior [2]. In this paper, inelastic stability characteristic equations in thirteen modes are derived for steel columns with plastic hinges on their ends of columns or beams associated with inelastic side-sway. The effective length factor *K* of a column in an inelastic stage is determined by solving the inelastic stability characteristic equations through a numerical process. The results show that, in some modes, the effective length factor K of steel columns in an inelastic stage is larger than that in an elastic stage. This means that the value of buckling load of steel columns in an inelastic stage is less than in an elastic stage. Therefore, if only elastic analysis is applied to the design of a steel column to determine its elastic stability, inelastic buckling could still occur after the first plastic hinge develops on the frame. To prevent such seismic instability, the design of a steel frame should incorporate both elastic stability and inelastic stability analysis [3~5].

THIRTEEN INELASTIC BUCKLING MODES FOR STEEL COLUMNS ON FRAME WITH INELASTIC SIDESWAY

Three major inelastic buckling types of substructures studied are shown in Fig. 1. The three types are divided into thirteen buckling modes in detail for inelastic stability analysis shown in Fig. 2. Type 1 involves a single column of the first story and consists of four modes: mode 1-01 to mode 1-04 respectively. In mode 1-01, a plastic hinge develops on the bottom end of a column while the top end remains in an elastic stage. Similarly, in mode 1-02, a plastic hinge develops on the top and bottom end of a column. In mode 1-03, a plastic hinge develops on the top end and a failure hinge occurs on the bottom end of a column. Finally, in mode 1-04, a failure hinge develops on the top and bottom ends of a column and on a beam end.

Type 2 inelastic buckling involves a side column and a side beam of the first story and also consists of four modes: mode 2-01 to mode 2-04 respectively. In mode 2-01, a plastic hinge develops on the top of column end, on the bottom of the column end and also on the beam end. Similarly, a plastic hinge occurs on the top and bottom end of a column, and a failure hinge occurs on the beam end in mode 2-02. In mode 2-03, a plastic hinge develops on the top end and a failure hinge develops on the bottom end of column, and a failure hinge develops on the beam end. Finally, in mode 2-04, a failure hinge occurs on the top and bottom ends of a column and on a beam end.

Type 3 inelastic buckling involves an internal column of the first story and two beams. Type 3 inelastic buckling consists of five modes: mode 3-01 to mode 3-05 respectively. In mode 3-01, there is a plastic hinge on both the top and the bottom of a column end and on both beam ends as well. Similarly, in mode 3-02 a plastic hinge develops on the top and bottom ends of a column and one side of a beam end while there is a

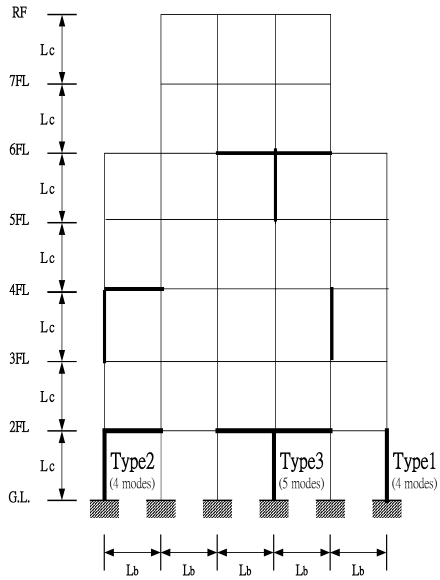

Fig. 1 Three major stability types for steel

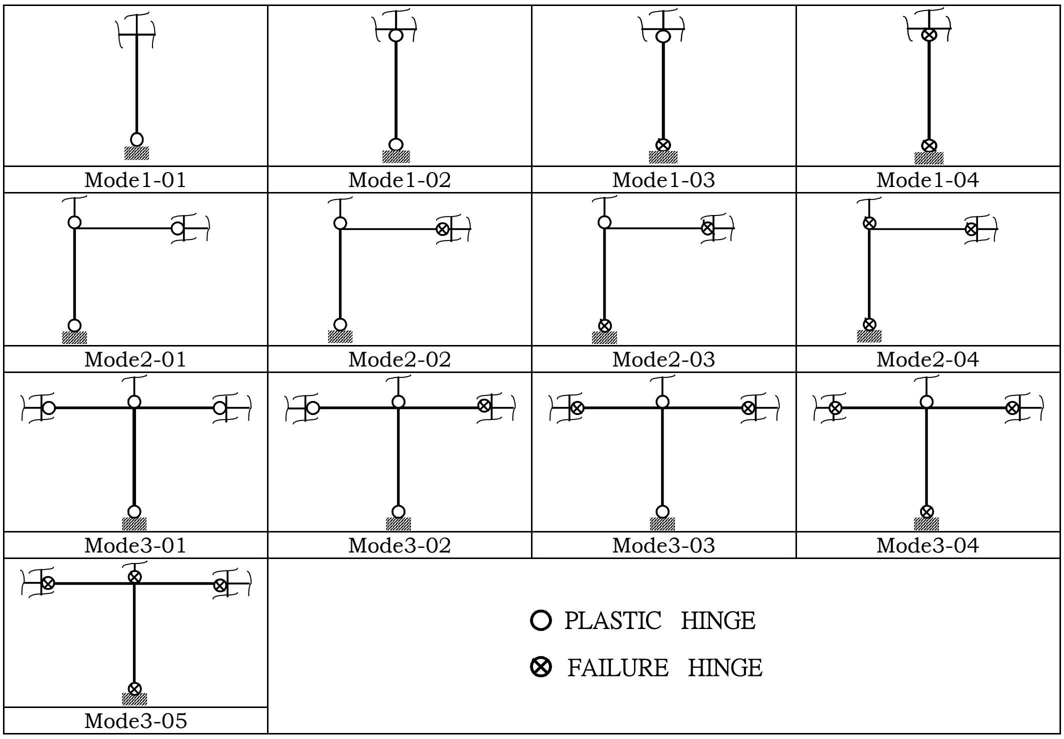

Fig. 2 Thirteen buckling modes for steel columns on frame in inelastic stage

failure hinge on another beam end. In mode 3-03, a plastic hinge occurs on the top and bottom ends and a failure hinge occurs on the end of both beams. In mode 3-04, a failure hinge occurs on the top end bottom ends of a column and a failure hinge occurs on the bottom end of a column. Finally, In mode 3-05, a failure hinge occurs on the top end and a failure hinge occurs on the beam ends and bottom end of a column.

The thirteen stability modes for a steel column studied are shown in Fig. 2.

THE OCCURRENCE OF PLASTIC HINGES AND FAILURE HINGES

In order to deal with the seismic stability analysis, several basic assumptions are followed: (1) The moment capacity of steel column is larger than that of steel beam:

$$(M_p)_{\text{column}} > (M_p)_{\text{beam}} \tag{1}$$

so that the plastic hinge will occur on the end of beam earlier than the end of column. (2) The distributing moment caused by vertical load and earthquake load at the bottom of a column is larger than at the top of the column. In other words, a plastic hinge will occur at the bottom end of a column before it occurs at its top. (3) A plastic hinge occurrs when the end moment is larger than yielding moment to approach the plastic moment:

$$M = M_p \quad \text{as} \quad \phi_y \le \phi \le \phi_p \tag{2}$$

(4) A failure hinge occurs when the bending curvature of a section caused by moment is larger than plastic curvature. At that time, the section cannot resist any moment but is still able to take shear force and axial force:

$$M = 0 \quad \text{as} \quad \phi > \phi_p \tag{3}$$

INELASTIC STABILITY ANALYSIS FOR TYPE 1 MODES

Mode 1-01 consists of a single column with flexure rigidity EI and length L, with a plastic hinge on the bottom end while the column top is side-sway with δ related to the bottom of column. For deriving the characteristic equation for buckling in inelastic stage, the governing equation of steel column can be established as:

$$EI\,y'' + P\,y = M_p - V\,x \quad (4)$$

where

$$V = \frac{M_{ba} - M_P - P\delta}{L} \quad (5)$$

rearranging Eq. (4) and solve for $y(x)$, the general solution for column shape function as:

$$y(x) = c_1 \cos kx + c_2 \sin kx + \frac{M_p}{k^2 EI} - \frac{M_{ba}}{k^2 EI\,L}x + \frac{M_P}{k^2 EI\,L}x + \frac{P\,\delta}{k^2 EI\,L}x \quad (6)$$

where

$$k^2 = \frac{P}{EI}$$

apply boundary conditions,

$y(0) = y'(0) = 0 \qquad y(L) = \delta$

into Eq. (6) and obtain:

$$y(x) = -\frac{M_p}{EIk^2}\cos kx + \left(\frac{M_{ba} - M_P - P\delta}{EI\,k^3 L}\right)\sin kx + \frac{M_p}{k^2 EI} - \frac{M_{ba}}{k^2 EI\,L}x + \frac{M_P}{k^2 EI\,L}x + \frac{P\delta}{k^2 EI\,L}x$$

Apply the slope-deflection relationship between end moment and deflection of column AB, thus

$$M_{ba} = \frac{2EI}{L}\left(-1.5\frac{\delta}{L}\right) + \frac{M_P}{2} \quad (7)$$

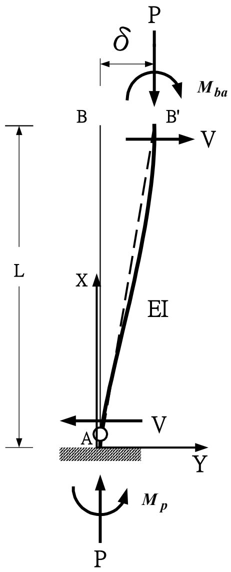

Fig. 3 Mode1-01

substitute Eq. (6) into Eq. (7), the characteristic equation for mode 1-01 is obtained with related to k as:

$$-\frac{M_P}{EIk^2}\cos kL + \left(\frac{-M_P}{2EIk^3 L} - \frac{3\delta}{k^3 L^3} - \frac{\delta}{kL}\right)\sin kL + \frac{3M_P}{2EIk^2} + \frac{3\delta}{k^2 L^2} = 0 \quad (8)$$

The value of k can be determined with numerical process and the effective length factor K can be calculated also as indicated in followed Eq. (15). The K values of mode 1-02 to mode1-04 will be able to be figured out with similar procedure.

INELASTIC STABILITY ANALYSIS FOR TYPE 2 MODES

Mode 2-01 consists of a side column with flexure rigidity $(EI)_c$ and length L_c and a side beam with flexure rigidity $(EI)_b$ and length L_b of the first story. There are three plastic hinges on the beam end and on the top end and bottom end of the column respectively. For deriving the characteristic equation for buckling on inelastic stage, the governing equation of column can be established as:

$$y'' + k^2 y = \frac{M_{PC}}{EI} - \frac{M_{PC}}{EI\ L_c} x + \frac{M_{ba}}{EI\ L_c} x + \frac{P\,\delta}{EI\ L_c} x \quad (9)$$

apply boundary conditions,

$$y\,(0) = 0 \qquad y\,(L_c) = \delta$$

into Eq. (9) and obtain:

$$\frac{1}{\sin kL_c}(M_{pc}\,k\cos kL_c - kM_{ab}) - \frac{M_{pc}}{L_c} + \frac{EI\,k^2\,\delta}{L_c} + \frac{1}{L_c} M_{ab} = 0 \quad (10)$$

From equilibrium of point B, the characteristic equation of k for mode 2-01 can be derived as:

$$M_{ba} = \frac{3(EI)_c}{L_c}\left(\frac{3M_{pc}}{2} + \frac{3(EI)_c}{L_c^2}\delta - \frac{M_{pb}}{2}\right) \Bigg/ \left(\frac{3(EI)_c}{L_c} + \frac{3(EI)_b}{L_b}\right) - \frac{3(EI)_c}{L_c^2}\delta - \frac{M_{pc}}{2} \quad (11)$$

The value of k can be determined with numerical process and the effective length factor K can be calculated also as indicated in followed Eq. (15). The K values of mode 2-02 to mode 2-04 will be able to be figured out with similar procedure.

INELASTIC STABILITY ANALYSIS FOR TYPE 3 MODES

Mode 3-04 consists of an internal column and two side beams. The flexure rigidity of column is $(EI)_c$ and the length of column is L_c. The flexure rigidity of beam is $(EI)_b$ and the length of beam is L_b.

Mode 3-04 with two failure hinges develops on the beam end and one failure hinge on the bottom end of the column. A plastic hinge develops on the top of column end. For deriving the characteristic equation for buckling on inelastic stage, the governing equation of column shape can be established as:

$$EI\ y'' + P\,y = \frac{M_{ba}}{L_c} x + \frac{P\,\delta}{L_c} x \quad (12)$$

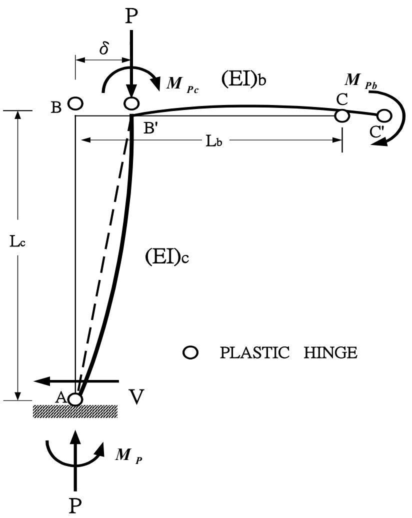

Fig. 4 Mode 2-01

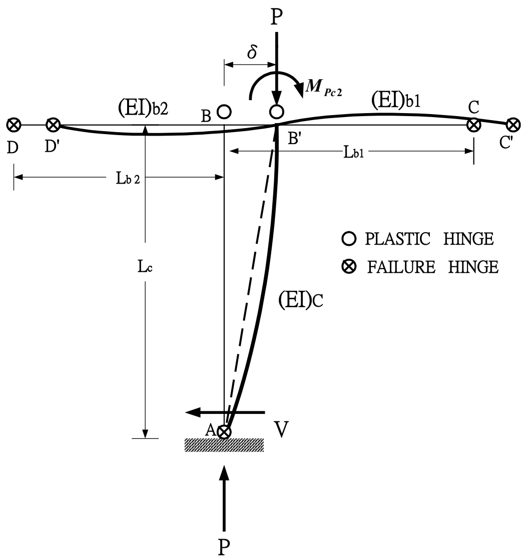

Fig. 5 Mode 3-04

Table 1 Characteristic equations of buckling related to k for thirteen inelastic buckling modes

Mode No.	Characteristic Equations of Buckling Related to k
Mode 1-01	$-\frac{M_P}{EI\ k^2}\cos kL + \left(\frac{-M_P}{2EI\ k^3 L} - \frac{3\delta}{k^3 L^3} - \frac{\delta}{kL}\right)\sin kL + \frac{3M_P}{2EI\ k^2} + \frac{3\delta}{k^2 L^2} = 0$
Mode 1-02	$kM_p\left(\frac{1-\cos kL}{\sin kL}\right) - \frac{EI\ k^2\delta}{L} = 0$
Mode 1-03	$\frac{kM_p}{\sin kL} - \frac{EI\ k^2\delta}{L} = \frac{M_p}{L}$
Mode 1-04	$\frac{k\delta}{\sin kL}\cos\frac{kL}{2} - \frac{\delta}{L} = 0$
Mode 2-01	$\frac{1}{\sin kL}(M_{pc}\, k\cos kL - kM_{ab}) - \frac{M_{pc}}{L_c} + \frac{EI\ k^2\delta}{L_c} + \frac{1}{L_c}M_{ab} = 0$ $M_{ab} = \frac{3(EI)_c}{L_c}\left(\frac{3M_{pc}}{2} + \frac{3(EI)_c}{L_c^2}\delta - \frac{M_{pb}}{2}\right)\Big/\left(\frac{3(EI)_c}{L_c} + \frac{3(EI)_b}{L_b}\right) - \frac{3(EI)_c}{L_c^2}\delta - \frac{M_{pc}}{2}$
Mode 2-02	$\frac{1}{\sin kL}(M_{pc}\, k\cos kL - kM_{ab}) - \frac{M_{pc}}{L_c} + \frac{EI\ k^2\delta}{L_c} + \frac{1}{L_c}M_{ab} = 0$ $M_{ab} = \frac{3(EI)_c}{L_c}\left(\frac{3M_{pc}}{2} + \frac{3(EI)_c}{L_c^2}\delta\right)\Big/\left(\frac{3(EI)_c}{L_c} + \frac{3(EI)_b}{L_b}\right) - \frac{3(EI)_c}{L_c^2}\delta - \frac{M_{pc}}{2}$
Mode 2-03	$\frac{1}{\sin kL}(kM_{ab}) - \frac{EI\ k^2\delta}{L_c} - \frac{1}{L_c}M_{ab} = 0$ $M_{ab} = \frac{3(EI)_c}{L_c}\left(M_{pc} + \frac{3(EI)_c}{L_c^2}\delta\right)\Big/\left(\frac{3(EI)_c}{L_c} + \frac{3(EI)_b}{L_b}\right) - \frac{3(EI)_c}{L_c^2}\delta$
Mode 2-04	$\frac{1}{\sin kL}(kM_{ab}) - \frac{EI\ k^2\delta}{L_c} - \frac{1}{L_c}M_{ab} = 0$ $M_{ab} = \frac{3(EI)_c}{L_c}\left(\frac{3(EI)_c}{L_c^2}\delta\right)\Big/\left(\frac{3(EI)_c}{L_c} + \frac{3(EI)_b}{L_b}\right) - \frac{3(EI)_c}{L_c^2}\delta$
Mode 3-01	$\frac{1}{\sin kL}(M_{pc}\, k\cos kL - kM_{ab}) - \frac{M_{pc}}{L_c} + \frac{EI\ k^2\delta}{L_c} + \frac{1}{L_c}M_{ab} = 0$ $M_{ab} = \frac{3(EI)_c}{L_c}\left(\frac{3M_{pc}}{2} + \frac{3(EI)_c}{L_c^2}\delta - M_{pb}\right)\Big/\left(\frac{6(EI)_c}{L_c} + \frac{3(EI)_b}{L_b}\right) - \frac{3(EI)_c}{L_c^2}\delta - \frac{M_{pc}}{2}$
Mode 3-02	$\frac{1}{\sin kL}(M_{pc}\, k\cos kL - kM_{ab}) - \frac{M_{pc}}{L_c} + \frac{EI\ k^2\delta}{L_c} + \frac{1}{L_c}M_{ab} = 0$ $M_{ab} = \frac{3(EI)_c}{L_c}\left(\frac{3M_{pc}}{2} + \frac{3(EI)_c}{L_c^2}\delta - \frac{M_{pb}}{2}\right)\Big/\left(\frac{6(EI)_c}{L_c} + \frac{3(EI)_b}{L_b}\right) - \frac{3(EI)_c}{L_c^2}\delta - \frac{M_{pc}}{2}$
Mode 3-03	$\frac{1}{\sin kL}(M_{pc}\, k\cos kL - kM_{ab}) - \frac{M_{pc}}{L_c} + \frac{EI\ k^2\delta}{L_c} + \frac{1}{L_c}M_{ab} = 0$ $M_{ab} = \frac{3(EI)_c}{L_c}\left(\frac{3M_{pc}}{2} + \frac{3(EI)_c}{L_c^2}\delta\right)\Big/\left(\frac{6(EI)_c}{L_c} + \frac{3(EI)_b}{L_b}\right) - \frac{3(EI)_c}{L_c^2}\delta - \frac{M_{pc}}{2}$
Mode 3-04	$\frac{1}{\sin kL}(kM_{ab}) - \frac{EI\ k^2\delta}{L_c} - \frac{1}{L_c}M_{ab} = 0$ $M_{ab} = \frac{3(EI)_c}{L_c}\left(M_{pc} + \frac{3(EI)_c}{L_c^2}\delta\right)\Big/\left(\frac{6(EI)_c}{L_c} + \frac{3(EI)_b}{L_b}\right) - \frac{3(EI)_c}{L_c^2}\delta$
Mode 3-05	$M_{ab} = \frac{3(EI)_c}{L_c}\left(\frac{3(EI)_c}{L_c^2}\delta\right)\Big/\left(\frac{6(EI)_c}{L_c} + \frac{3(EI)_b}{L_b}\right) - \frac{3(EI)_c}{L_c^2}\delta$

Table 2 Column effective length factor K for elastic and inelastic stage

Steel Frame Stability Mode	Column Effective Length Factor			
	Inelastic Stage K			Elastic Stage K
	δ = (5/1000)*L	δ = (10/1000)*L	δ = (20/1000)*L	–
Mode 1-01	1.3383	1.2736	1.1835	1.4552
Mode 1-02	1.7336	1.6748	1.2822	1.4552
Mode 1-03	1.6350	1.2230	1.0987	1.4552
Mode 1-04	–	–	–	1.4552
Mode 2-01	1.5379	1.3698	1.0620	1.7636
Mode 2-02	1.5300	1.3204	1.0852	1.7636
Mode 2-03	1.2334	1.0562	0.9871	1.7636
Mode 2-04	0.9260	0.9260	0.9260	1.7636
Mode 3-01	1.0001	1.1164	0.9660	1.4552
Mode 3-02	0.9999	1.1689	0.9817	1.4552
Mode 3-03	1.5518	1.2272	0.9958	1.4552
Mode 3-04	1.0924	0.9790	0.9320	1.4552
Mode 3-05	0.8873	0.8873	0.8873	1.4552

Rearranging Eq. (12) and solve for $y(x)$, the general solution for column shape function as:

$$y(x) = c_1 \cos kx + c_2 \sin kx + \frac{M_{ba}}{k^2\, EI\, L_c} x + \frac{P\,\delta}{k^2\, EI\, L_c} x \qquad (13)$$

Apply boundary condition:

$$y(0) = 0 \qquad y(L_c) = \delta$$

and from equilibrium of point B, the characteristic equation for mode 3-04 is derived as:

$$M_{ab} = \frac{3(EI)_c}{L_c}\left(M_{pc} + \frac{3(EI)_c}{L_c^2}\delta\right) \Bigg/ \left(\left(\frac{6(EI)_c}{L_c} + \frac{3(EI)_b}{L_b}\right) - \frac{3(EI)_c}{L_c^2}\delta\right) \qquad (14)$$

ILLUSTRATIONS

For determining the value of effective length factor K, the steel column section as H700 × 300 × 13 × 24 and length L_c = 5m is considered. The flexural rigidity EI = 402816.85kN-m^2 and plastic moment capacity for M_p = 1596.32kN-m is calculated for steel column. Similarly, the section of beam is H500 × 300 × 11 × 18 and length L_b = 5m is illustrated. The flexural rigidity EI = 141847.16kN-m^2 is calculated and plastic moment capacity for beam is M_p = 797.23kN-m. In this study, three different sidesway values as 5L/1000, 10L/1000, and 20L/1000 respectively are considered to determine the effective length factor K for member on inelastic stage. The characteristic equations of k for thirteen buckling modes are derived and shown in Table 1, then the value for

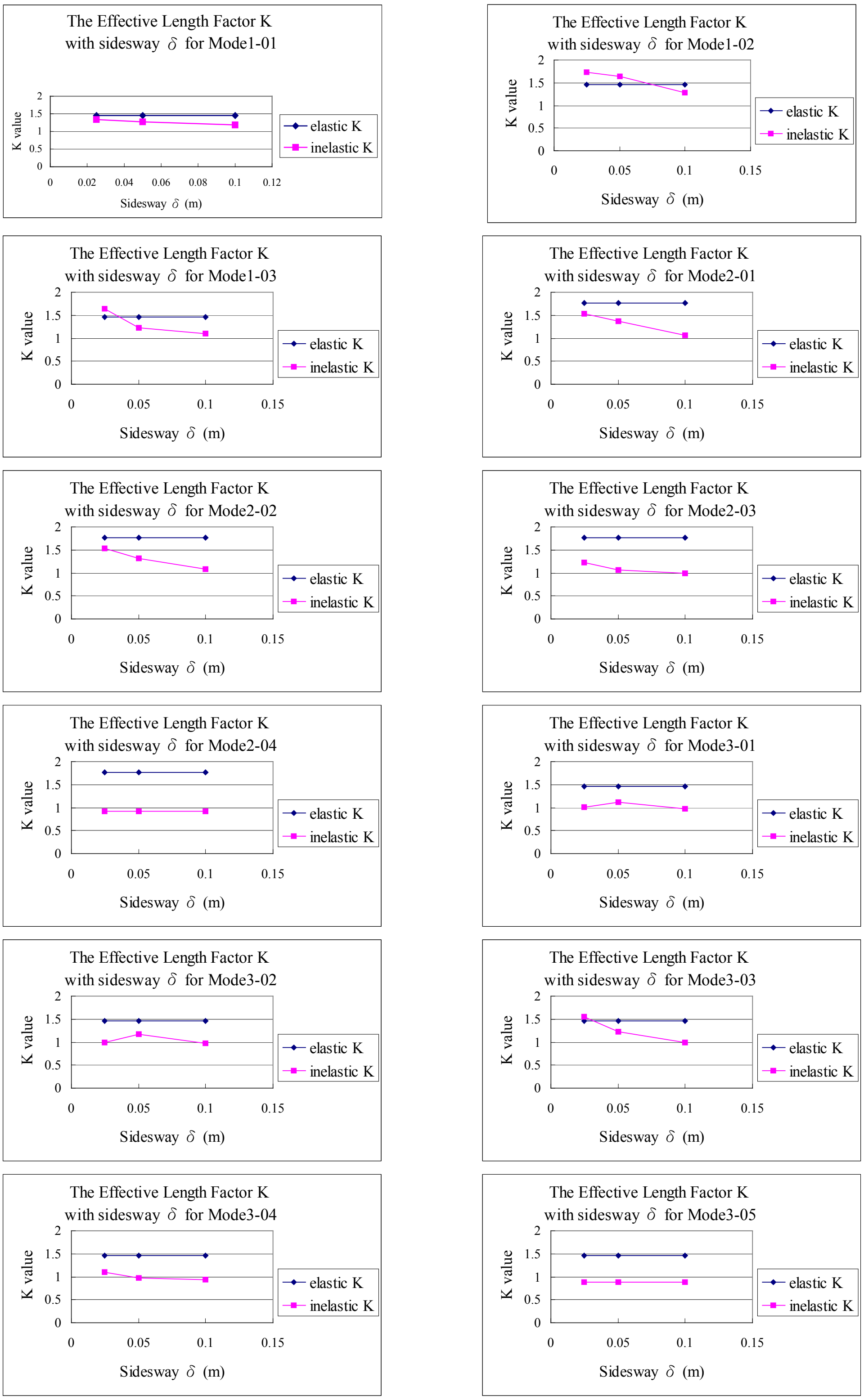

Fig. 6 The inelastic and elastic effective length factor K

$(EI)_c$, $(EI)_b$, L_c, L_b, M_{pc} and M_{pb} are substitute into the characteristic equations. By numerical process, the value of k can be determined. One can finally get the effective length factor K for member on inelastic stage by Eq. (15).

$$K = \sqrt{\frac{P_e}{P_{cr}}} = \frac{\pi}{k\,L} \qquad (15)$$

The K value for each mode and for different sidesway on inelastic stage is shown in Table 2. Notice that the K value on elastic stage shown is calculated by Eq. (16):

$$K = \frac{20 - \varphi_m}{20}\sqrt{1+\varphi_m} \quad \text{for} \quad \varphi_m \le 2$$
$$K = 0.9\sqrt{1+\varphi_m} \quad \text{for} \quad \varphi_m > 2 \qquad (16)$$

RESULTS ANALYSIS

From the results of analysis, the eigenvalue of k related to characteristic equation for steel column in mode1-04 is zero. In other words, mode1-04 is an instability mode. Because the top end and bottom end of column for mode1-04 is with failure hinges and with sidesway, the mode is theoretical instability. This result justifies the correctness of derivation and numerical process in this study. For the cases of mode1-02 with sidesway of $5L/1000$ and $10L/1000$, mode1-03 with sidesway of $5L/1000$ and mode 3-03 with sidesway of $5L/1000$, the values of inelastic effective length factor K is larger than elastic effective length factor. This means that the value of buckling load of those cases of steel column on inelastic stage is less than that on elastic stage. Therefore, if the steel column is designed just by elastic analysis and is considered elastic stability analysis only, the inelastic buckling would happen after the first plastic hinge occur on the frame. In order to prevent the seismic instability happened, the both analyses dealt with elastic stability and inelastic stability analysis should be well carried through for the safety of the steel frame.

CONCLUSION

After this study the following conclusion remarks can be drawn: (1) The value of effective length factor K for a steel column in an inelastic stage is different from that in an elastic stage. (2) For the cases of mode 1-02 with side-sway of $5L/1000$ and $10L/1000$, mode 1-03 with side-sway of $5L/1000$ and mode 3-03 with side-sway of $5L/1000$, the values of inelastic effective length factor K are larger than the elastic effective length factor. This means that the value of buckling load of those cases involving a steel column in an inelastic stage is less than for those cases in an elastic stage. (3) The seismic lateral resistant capacity for steel frame evaluation will be not sufficiently safe if the steel columns of a frame are designed only by dealing with elastic stability analysis without inelastic stability analysis. To prevent seismic instability, both elastic and inelastic stability analysis should be considered. (4) The proposed substructure buckling in thirteen modes described in this study can be extended for the further application. (5) Inelastic stability analysis should be investigated more information for seismic capacity assessment of a multistory steel frame.

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