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Real Time Conditional Simulation of Earthquake Ground Motion

Tadanobu Sato [1)] Hirofumi Imabayashi [2)]

1) Professor, Disaster Prevention Research Institute, Kyoto University.
2) Engineer, Toda Cooperation.

ABSTRACT

A method is presented for simulating earthquake ground motions at local-field points under the condition that the recorded motions are specified at several locations and the stochastic characteristics of the field also are designated. The simulated motions at observation points coincide with observed records. The state equation expressing the spatial and temporal field of earthquake ground motion compatible with the modeled stochastic field is derived by using the autoregressive process. The Kalman filtering technique is used to identify the best adaptive estimator of a stochastic field from observations made at discrete spatial and temporal points. To take into account the nonstationarity and inhomogeneity of earthquake ground motions, the algorithm to identify the stochastic characteristics of the field from observed earthquake motions is also developed and introduced to the conditional simulation algorithm. This analytical method is very promising for conditional simulation of earthquake motion at unobserved locations.

INTRODUCTION

Earthquake ground motions vary in time and space. Various methods for simulating space-time correlated ground motion have been proposed. The one most widely used is a probabilistic approach based on the cross-correlation function or cross-spectrum. The theory of random fields and its application to digital simulation of earthquake ground motions are well established [1]. As substantial databases of earthquake ground motions based on the dense strong motion arrays have been formed, the stochastic characteristics of the space-time correlation of earthquake motions are modeled by the function of the separation distance between two points, the traveling time of the wave, the frequency, and the wave number. Waves simulated by the random field theory, however, are only sample waves.

Conditional simulation methods must be devised that allows realization of a sample field at an unobserved location which satisfies the properties of a stochastic field and is compatible with measured values at observed locations. To obtain this kind of simulation a method that used closed form solutions of conditional probability functions for Fourier coefficients [2], nonstochastic conditional simulation [3], and application of the Kriging method [4,5] has been developed. In these analyses, as all time histories of propagating waves at observation points must be known apriori, are not applicable to the real-time conditional simulation of earthquake motions. The use of observed real-time earthquake motions obtained from existing array observation systems to simulate earthquake motions at several unobserved locations on real-time is main concern of this paper.

SPACE-TIME EQUATION OF EARTHQUAKE MOTION

The temporal and spatial fields of earthquake motion are assumed to be expressed by a spatially correlated autoregressive process

$$\mathbf{z}_t = -\mathbf{A}_1\,\mathbf{z}_{t-1} - \mathbf{A}_2\,\mathbf{z}_{t-2} - \cdots - \mathbf{A}_q\,\mathbf{z}_{t-q} + \Gamma\,\mathbf{w}_t \tag{1}$$

in which $\mathbf{z}_t$ is a vector composed of n-dimensional earthquake motion at time t

$$\mathbf{z}_t = \{\, g_1(t),\ g_2(t),\ \cdots,\ g_m(t)\,\}^T \tag{2}$$

When we define the following vector

$$\mathbf{Z}_{t-1} = \{\, \mathbf{z}_{t-1}^T,\ \mathbf{z}_{t-2}^T,\ \cdots,\ \mathbf{z}_{t-q}^T\,\}^T \tag{3}$$

Eq. (1) can be rewritten

$$\mathbf{z}_t = -\Phi\,\mathbf{Z}_{t-1} + \Gamma\,\mathbf{w}_t\ ,\quad \Phi = [\,\mathbf{A}_1,\ \mathbf{A}_2,\ \cdots,\ \mathbf{A}_q\,] \tag{4}$$

in which $\mathbf{w}_t$ is the system noise with covariance matrix $\mathbf{Q}_t = E[(\mathbf{w}_t - \overline{\mathbf{w}}_t)(\mathbf{w}_t - \overline{\mathbf{w}}_t)^T]$ and $\overline{\mathbf{w}}_t$ the mean value of noise at time t. When the posterior best estimator of $\mathbf{Z}_{t-1}$ is given by $\hat{\mathbf{Z}}_{t-1}$, the apriori best estimator of $\mathbf{z}_t$ is calculated as

$$\overline{\mathbf{z}}_t = -\Phi\ \hat{\mathbf{Z}}_{t-1} + \Gamma\ \overline{\mathbf{w}}_t \tag{5}$$

Defining the difference between $\mathbf{z}_{t-k}$ and $\overline{\mathbf{z}}_{t-k}$ for arbitrary k

$$\tilde{\mathbf{z}}_{t-k} = \mathbf{z}_{t-k} - \overline{\mathbf{z}}_{t-k} \tag{6}$$

The covariance matrix of $\mathbf{z}_t$ is given by

$$\mathbf{M}_t = E[\,\tilde{\mathbf{z}}_t\ \tilde{\mathbf{z}}_t^T\,] = \Phi\,\mathbf{P}_{t-1}\,\Phi^T + \Gamma\,\mathbf{Q}_t\,\Gamma^T \tag{7}$$

in which $\mathbf{P}_{t-1}$ is the system covariance matrix at time $t-1$. If the earthquake motion is assumed to be a stochastic field that is homogeneous in time and stationary in space, this matrix is defined as

$$\mathbf{P}_{t-1} = E[\,\tilde{\mathbf{Z}}_{t-1}\ \tilde{\mathbf{Z}}_{t-1}^T\,] = \begin{bmatrix} \mathbf{R}(0) & \mathbf{R}(-1) & \cdots & \mathbf{R}(1-q) \\ \mathbf{R}(1) & \mathbf{R}(0) & \cdots & \mathbf{R}(2-q) \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{R}(q-1) & \mathbf{R}(q-2) & \cdots & \mathbf{R}(0) \end{bmatrix} \tag{8}$$

in which $\tilde{\mathbf{Z}}_{t-1} = \mathbf{Z}_{t-1} - \hat{\mathbf{Z}}_{t-1}$, $\hat{\mathbf{Z}}_{t-1}$ is the posterior best estimator of $\mathbf{Z}_{t-1}$, and $\mathbf{R}(p)$ the cross correlation matrix with dimension $(n \times n)$ defined by

$$\mathbf{R}(p) = E[\,\tilde{\mathbf{z}}_{t-k}\ \tilde{\mathbf{z}}_{t-l}^T\,]\ ,\quad (p = l - k) \tag{9}$$

The coefficient matrices $\mathbf{A}_i$ of the autoregressive process can be calculated by use of the cross correlation function of $\mathbf{z}_t$. Subtracting Eq. (5) from Eq. (1) and taking its transverse then multiplying by $\tilde{\mathbf{z}}_{t-k}$ $(k = 1, 2, \ldots, q)$ from the left;

$$E[\,\tilde{\mathbf{z}}_{t-k}\ \tilde{\mathbf{z}}_t^T\,] = -E[\,\tilde{\mathbf{z}}_{t-k}\ \tilde{\mathbf{z}}_{t-1}^T\,]\,\mathbf{A}_1^T - E[\,\tilde{\mathbf{z}}_{t-k}\ \tilde{\mathbf{z}}_{t-2}^T\,]\,\mathbf{A}_2^T - \cdots - E[\,\tilde{\mathbf{z}}_{t-k}\ \tilde{\mathbf{z}}_{t-q}^T\,]\,\mathbf{A}_q^T \quad (10)$$

Taking into account Eq. (9), Eq. (10) is transferred to the following simultaneous equation to define the coefficient matrices of the autoregressive process in Eq. (1)

$$-\begin{bmatrix} \mathbf{R}(0) & \mathbf{R}(-1) & \cdots & \mathbf{R}(1-q) \\ \mathbf{R}(1) & \mathbf{R}(0) & \cdots & \mathbf{R}(2-q) \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{R}(q-1) & \mathbf{R}(q-2) & \cdots & \mathbf{R}(0) \end{bmatrix} \begin{bmatrix} \mathbf{A}_1^T \\ \mathbf{A}_2^T \\ \vdots \\ \mathbf{A}_q^T \end{bmatrix} = \begin{bmatrix} \mathbf{R}(1) \\ \mathbf{R}(2) \\ \vdots \\ \mathbf{R}(q) \end{bmatrix} \quad (11)$$

Once the cross correlation matrix defined by Eq. (9) and the observed time series of earthquake motion from time $t-q$ to $t-1$ are given, earthquake motion at time t can be simulated by Eq. (1).

CONDITIONAL SIMULATION OF EARTHQUAKE MOTION

Consider the case in which the total number of points for which the earthquake ground motion needed is n, but records of earthquake motion are obtained only at m ($m < n$) observation points as defined by

$$\mathbf{y}_t = \{\, g_1(t),\ g_2(t),\ \cdots,\ g_m(t)\,\} \quad (12)$$

The observation equation then is

$$\mathbf{y}_t = [\,\mathbf{H}\,]\ \mathbf{z}_t\ , \quad \mathbf{H} = [\,\mathbf{I}\quad \mathbf{0}\,] \quad (13)$$

in which $\mathbf{H}$ is the observation matrix with the dimension ($m \times n$) and $\mathbf{I}$ the unit matrix with the dimension ($m \times m$). This is a special case in that the observation vector $\mathbf{y}_t$ and the vector $\mathbf{z}_t$ to be estimated have the same physical value [6]. The covariance matrix of the observation noise, $\mathbf{S}_t$, is therefore assumed to be the zero matrix

$$\mathbf{S}_t = \mathbf{0} \quad (14)$$

The Kalman filtering technique [7] is used to obtain the posterior best estimator of earthquake motion at time t conditioned by the observation equation. Because the apriori best estimator and its covariance matrix are given by Eqs. (5) and (7), the posterior best estimator of $\mathbf{z}_t$ and its covariance matrix $\mathbf{P}_t$ are

$$\hat{\mathbf{z}}_t = \bar{\mathbf{z}}_t + \mathbf{K}_t\,(\,\mathbf{y}_t - \mathbf{H}\,\bar{\mathbf{z}}_t\,) \quad (15)$$

$$\mathbf{P}_t = \mathbf{M}_t - \mathbf{K}_t\,\mathbf{H}\,\mathbf{M}_t \quad (16)$$

in which $\mathbf{K}_t$ is a Kalman gain at time t and is defined as

$$\mathbf{K}_t = \mathbf{M}_t\ \mathbf{H}^T(\,\mathbf{H}\,\mathbf{M}_t\,\mathbf{H}^T + \mathbf{S}_t\,)^{-1} \quad (17)$$

Partitioning the matrix $\mathbf{M}_t$ into the observed and unobserved parts

$$\mathbf{M}_t = \begin{bmatrix} \mathbf{M}_{t;\,m,\,m} & \mathbf{M}_{t;\,m,\,n-m} \\ \mathbf{M}_{t;\,n-m,\,m} & \mathbf{M}_{t;\,n-m,\,n-m} \end{bmatrix} \quad (18)$$

and substituting the $\mathbf{H}$ matrix given in Eq. (13) and Eq. (18) into Eqs. (16) and (17), the Kalman gain and covariance matrix $\mathbf{P}_t$ yield

$$\mathbf{K}_t = \begin{bmatrix} \mathbf{I} \\ \mathbf{M}_{t;\,n-m,\,m}\ \ \mathbf{M}_{t;\,m,\,m}^{-1} \end{bmatrix} \quad (19)$$

$$\mathbf{P}_t = \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{M}_{t;\,n-m,\,n-m} - \mathbf{M}_{t;\,m,\,n-m}^T\,\mathbf{M}_{t;\,m,\,m}^{-1}\,\mathbf{M}_{t;\,m,\,n-m} \end{bmatrix} \quad (20)$$

Substituting Eqs. (19) and (20) into Eq. (15) the best posterior estimator of earthquake motion is

$$\hat{\mathbf{z}}_t = \begin{Bmatrix} \hat{\mathbf{z}}_{t;\,m} \\ \hat{\mathbf{z}}_{t;\,n-m} \end{Bmatrix} = \begin{Bmatrix} \mathbf{y}_t \\ \bar{\mathbf{z}}_{t;\,n-m} + \mathbf{M}_{t;\,m,\,n-m}^T\mathbf{M}_{t;\,m,\,m}^{-1}(\mathbf{y}_t - \bar{\mathbf{z}}_{t;\,m}) \end{Bmatrix} \quad (21)$$

These results show that the posterior best estimators of earthquake motion at observation points are identical to the observed values and that the components of covariance matrix become zero.

To obtain sample earthquake motion at an unobserved point we must add a sample fluctuation to the posterior best estimator of earthquake motion. The sample field is simulated by

$$\mathbf{z}_{t;\,n-m} = \hat{\mathbf{z}}_{t;\,n-m} + \mathbf{e} \tag{22}$$

in which $\mathbf{e}$ is the sample error function vector simulated by multiply-correlated process with a zero mean vector, and covariance $\mathbf{P}_{t;n-m,n-m}$ being defined by

$$\begin{aligned}\mathbf{P}_{t;\,n-m,\,n-m} &= E\,[\,(\mathbf{z}_{t;\,n-m} - \hat{\mathbf{z}}_{t;\,n-m})\,(\mathbf{z}_{t;\,n-m} - \hat{\mathbf{z}}_{t;\,n-m})^T\,] \\ &= \mathbf{M}_{t;\,n-m,\,n-m} - \mathbf{M}^T_{t;\,m,\,n-m}\mathbf{M}^{-1}_{t;\,m,\,m}\mathbf{M}_{t;\,m,\,n-m}\end{aligned} \tag{23}$$

By factoring matrix $\mathbf{P}_{t;n-m,n-m}$ the error sample function is

$$\mathbf{e} = \mathbf{L}_t\,\mathbf{N}\;, \qquad \mathbf{P}_{t;\,n-m,\,n-m} = \mathbf{L}_t\,\mathbf{L}_t^T \tag{24}$$

in which $\mathbf{N}$ is an $n - m$ dimensional sample noise vector generated by the standard normal distribution density function.

STOCHASTIC MODEL OF EARTHQUAKE MOTION

We assume that earthquake motion at point i and time t is expressed by the cross-correlated m stochastic process $u_i(t)$ ($i = 1, 2, \ldots, m$) [1] as

$$u_i(t) = \sum_{p=1}^{i}\sum_{n=1}^{N} \left| H_{ip}(\omega_n\sqrt{\Delta\omega}) \right| \sqrt{2}\,\cos\,\{\,\omega_n t + \theta_{ip} + \phi_{pn}\,\} \tag{25}$$

in which ϕ_{pn} are mutually independent uniformly distributed variables over $-\pi$ to π and

$$\omega_n = n\cdot\Delta\omega = \frac{2\,n\,\pi}{T} \tag{26}$$

in which T is the duration of earthquake motion. $H_{ip}(\omega_n)\sqrt{\Delta\omega}$ is obtained by factoring the cross spectrum S_{ij} (d_{ij} , ω_n) defined in the field as

$$\begin{bmatrix} S_{11}(d_{11},\omega_n)\Delta\omega & \cdots & S_{1m}(d_{ij},\omega_n)\Delta\omega \\ \vdots & \ddots & \vdots \\ S_{m1}(d_{11},\omega_n)\Delta\omega & \cdots & S_{mm}(d_{ij},\omega_n)\Delta\omega \end{bmatrix} = \begin{bmatrix} H^*_{11}(\omega_n)\sqrt{\Delta\omega} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ H^*_{m1}(\omega_n)\sqrt{\Delta\omega} & \cdots & H^*_{mm}(\omega_n)\sqrt{\Delta\omega} \end{bmatrix} \cdot \begin{bmatrix} H_{11}(\omega_n)\sqrt{\Delta\omega} & \cdots & H_{m1}(\omega_n)\sqrt{\Delta\omega} \\ \vdots & \ddots & \vdots \\ 0 & \cdots & H_{mm}(\omega_n)\sqrt{\Delta\omega} \end{bmatrix} \tag{27}$$

and θ_{ip} is calculated by

$$\theta_{ip} = \tan^{-1}\left\{ \frac{\mathrm{Im}\,[H_{ip}(\omega_n)\sqrt{\Delta\omega}]}{\mathrm{Re}\,[H_{ip}(\omega_n)\sqrt{\Delta\omega}]} \right\} \tag{28}$$

The cross spectrum is assumed to be given by the equation

$$S_{ij}(d_{ij},\omega_n)\,\Delta\omega = S_T(\omega_n)\,\Delta\omega\cdot\exp\left(\frac{-\,i\,\omega_n d_{ij}}{c}\right)\cdot\exp\left(\frac{-\,\alpha\omega_n\,|\,d_{ij}\,|}{2\pi c}\right) \tag{29}$$

in which d_{ij} is the relative distance between points i and j, c the propagating wave speed, and $S_T(\omega_n)$ the discretized homogeneous and stationary power spectrum density function at the circular frequency of ω_n and is defined by the following equation, provided the discrete Fourier cos and sin series amplitudes of

a_n and b_n are given at a certain point in the concerned field,

$$S_T(\omega_n)\,\Delta\omega = \frac{a_n^2 + b_n^2}{2} \tag{30}$$

To make the conditional simulation of earthquake ground motions, the cross correlation function must be defined in the field obtained by inverse Fourier transform of the cross spectrum;

$$\begin{aligned} R_{ij}(d_{ij},\tau) &= \frac{1}{2}\int_{-\infty}^{\infty} S_{ij}(d_{ij},\omega)\exp(i\omega\tau)\,dw \\ &= \frac{1}{2}\sum_{n=1}^{N}(a_n^2 + b_n^2)\exp\left(\frac{-\alpha\,\omega_n\,|\,d_{ij}\,|}{2\pi c}\right) \\ &\quad\cdot\cos\left\{\omega_n\left(\tau - \frac{d_{ij}}{c}\right)\right\} \end{aligned} \tag{31}$$

in which τ is the relative time delay of earthquake motion between points i and j.

Equation (31) is intended only to illustrate the procedure, but it does show some characteristics of the empirical correlation functions obtained from dense accelerograph array data. Figure 1 shows the simulated sample fields of a one-dimensional wave propagation with velocity of c =1000m/s and α = 0.02 from Eq. (25). Each point is separated by 100m. In the simulation earthquake ground motion at the left edge point is assumed to be given and its discrete Fourier spectrum amplitude calculated (Fig. 2), after which the waveforms of the remaining points can be simulated.

IDENTIFICATION OF AUTOREGRESSIVE PROCESS

Identification of Observed Parts

So far we have assumed that the temporal and spatial stochastic characteristics are defined in the concerned

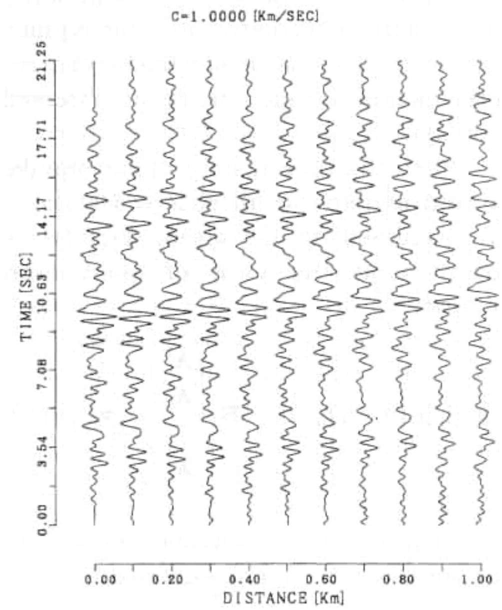
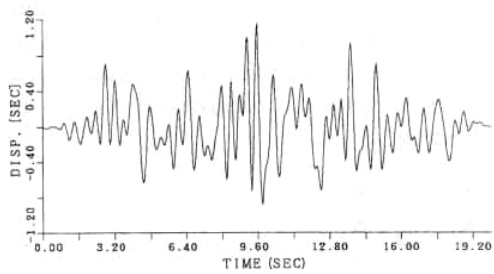
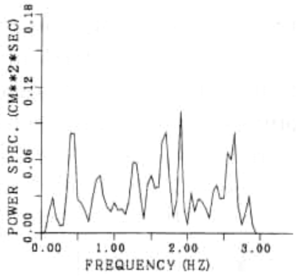


Fig. 1 Sample earthquake ground motions simulated by Eq. (25)



(a) time history



(b) Fourier spectrum

Fig. 2 Input earthquake motion into the time space field

field apriori. Because of nonstationarity and nonhomogeneity of earthquake ground motions these stochastic characteristics must be identified from observed motions.

When the time history of earthquake ground motions at all observed stations are assumed to be given, Eq. (4) is rewritten in the form of observation equation

$$\{ g_1(t), g_2(t), \cdots, g_m(t) \} = \mathbf{H}_t \begin{bmatrix} \mathbf{A}_1^T \\ \mathbf{A}_2^T \\ \vdots \\ \mathbf{A}_q^T \end{bmatrix}_t + \mathbf{v}_t \quad (32)$$

in which $\mathbf{v}_t$ is the observation noise and the observation matrix is defined by

$$\mathbf{H}_t = \mathbf{Z}_{t-1}^T = -\{ \mathbf{z}_{t-1}^T, \mathbf{z}_{t-2}^T, \cdots, \mathbf{z}_{t-q}^T \} \quad (33)$$

in which $\mathbf{z}_t$ is a vector composed of m-dimensional observed earthquake motion at time t; $\mathbf{z}_t = \{ g_1(t), g_2(t), \ldots, g_m(t) \}$. The state space description of the coefficient matrices of the autoregressive process is given by

$$\begin{bmatrix} \mathbf{A}_1^T \\ \mathbf{A}_2^T \\ \vdots \\ \mathbf{A}_q^T \end{bmatrix} = [\mathbf{I}] \begin{bmatrix} \mathbf{A}_1^T \\ \mathbf{A}_2^T \\ \vdots \\ \mathbf{A}_q^T \end{bmatrix}_{t-1} \quad (34)$$

To apply the Kalman filtering algorithms for identifying autoregressive process the coefficient matrix appeared in Eqs. (32) and (34) is rearranged as a matrix composed of column vectors $\mathbf{a}_{k,t}$ ($k = 1, 2, \ldots, m$)

$$\begin{bmatrix} \mathbf{A}_1^T \\ \mathbf{A}_2^T \\ \vdots \\ \mathbf{A}_q^T \end{bmatrix} = [\mathbf{a}_{1,t} \quad \mathbf{a}_{2,t} \quad \cdots \quad \mathbf{a}_{m,t}] \quad (35)$$

The state space and observation equations can be rewritten

$$\mathbf{a}_{k,t} = [\mathbf{I}]\, \mathbf{a}_{k,t-1} \qquad g_k(t) = \mathbf{H}_t\, \mathbf{a}_{k,t} + \mathbf{v}_{k,t} \quad (36)$$

in which

$$\mathbf{H}_t = -\{ \mathbf{z}_{t-1}^T, \mathbf{z}_{t-2}^T, \cdots, \mathbf{z}_{t-q}^T \}$$

The algorithm of the Kalman filtering for Eq. (36) are formulated as developed in [8]

(1) Initial conditions:

$$\hat{\mathbf{a}}_{k,0} = \mathbf{a}_0, \quad \mathbf{P}_{k,0} = \mathbf{P}_0, \quad \mathbf{R} = \mathbf{R}_0, \quad t = 0$$

(2) Time step increase: $t = t + 1$

(3) Predicted state estimation:

$$\bar{\mathbf{a}}_{k,t} = \hat{\mathbf{a}}_{k,t-1}, \qquad \mathbf{M}_{k,t} = \mathbf{P}_{k,t-1}$$

(4) Kalman gain:

$$\mathbf{K}_{k,t} = \mathbf{M}_{k,t}\, \mathbf{H}_t^T\, (\mathbf{H}_t\, \mathbf{M}_{k,t}\, \mathbf{H}_t^T + \mathbf{R})^{-1}$$

(5) Corrected state estimation:

$$\hat{\mathbf{a}}_{k,0} = \bar{\mathbf{a}}_{k,t} + \mathbf{K}_{k,t}\, (g_k(t) - \mathbf{H}_t\, \bar{\mathbf{a}}_{k,t})$$

(6) Estimation of error covariance matrix:

$$\mathbf{P}_{k,t} = \mathbf{M}_{k,t} - \mathbf{K}_{k,t}\, \mathbf{H}_t\, \mathbf{M}_{k,t}$$

Through the steps from (1) to (6) the posterior best estimator of coefficient matrices of autoregressive process is calculated.

Identification of Unobserved Parts

To identify the coefficient matrices of autoregressive process related to unobserved parts the cross-correlation function of the concerned field must be given. Because we assume the form expressed by Eq. (31) for cross-correlation function, identification of the parameters α and c from observed earthquake ground motions are enough to define the cross-correlation function. Once this function identified Eq. (11) is rewritten as

$$\mathbf{R}\, \mathbf{A} = \mathbf{r} \quad (37)$$

in which $\mathbf{R}$, $\mathbf{A}$ and $\mathbf{r}$ are the matrices rearranged their components and partitioned into the observed and unobserved parts as

$$\mathbf{R} = \begin{bmatrix} \mathbf{R}^{oo} & \mathbf{R}^{ou} \\ \mathbf{R}^{uo} & \mathbf{R}^{uu} \end{bmatrix} \quad \mathbf{A} = \begin{bmatrix} \mathbf{A}^{oo} & \mathbf{A}^{ou} \\ \mathbf{A}^{uo} & \mathbf{A}^{uu} \end{bmatrix} \quad \mathbf{r} = \begin{bmatrix} \mathbf{r}^{0} \\ \mathbf{r}^{u} \end{bmatrix} \tag{38}$$

The partitioned matrix $\mathbf{A}^{oo}$ can be identified by using observed earthquake ground motions as mentioned in the previous section. The components of auto-regressive matrices related to unobserved points can be determined by solving following equations

$$\left.\begin{aligned} \sum_{j=1}^{m\times q} \mathbf{R}_{ij}^{oo}\,\mathbf{A}_{jl}^{oo} + \sum_{j=1}^{(n-m)\times q} \mathbf{R}_{ij}^{ou}\,\mathbf{A}_{jl}^{uo} = \mathbf{r}_{il}^{o} \\ \sum_{j=1}^{m\times q} \mathbf{R}_{kj}^{uo}\,\mathbf{A}_{jl}^{oo} + \sum_{j=1}^{(n-m)\times q} \mathbf{R}_{kj}^{uu}\,\mathbf{A}_{jl}^{uo} = \mathbf{r}_{kl}^{u} \end{aligned}\right\} \tag{39}$$

$$\left.\begin{aligned} \sum_{j=1}^{m\times q} \mathbf{R}_{ij}^{oo}\,\mathbf{A}_{js}^{oo} + \sum_{j=1}^{(n-m)\times q} \mathbf{R}_{ij}^{ou}\,\mathbf{A}_{js}^{uu} = \mathbf{r}_{is}^{o} \\ \sum_{j=1}^{m\times q} \mathbf{R}_{kj}^{uo}\,\mathbf{A}_{js}^{ou} + \sum_{j=1}^{(n-m)\times q} \mathbf{R}_{kj}^{uu}\,\mathbf{A}_{js}^{uu} = \mathbf{r}_{ks}^{u} \end{aligned}\right\} \tag{40}$$

in which (l = 1, 2, …, m), (s = m + 1, m + 2, …, n), (i = 1, 2, …, $m \times q$) and (k = 1, 2, …, $(n-m) \times q$). Solving Eq. (40) the unknown values of $\mathbf{A}_{jl}^{ou}$ and $\mathbf{A}_{jl}^{uu}$ can be determined. As can be seen from Eq. (39) the number of unknown values is $(n-m) \times q$ but the number of equation is $n \times q$. We can apply the least square estimation algorithm to define the most appropriate values of $\mathbf{A}_{jl}^{uo}$.

NUMERICAL EXAMPLES

To verify the applicability of the autoregressive process for simulating multi-correlated earthquake motions, we assume that the sample earthquake motions shown in Fig. 1 are observed at all points up to time $t-1$ and that earthquake motions at time t are simulated by Eq. (4) with no system noise. The coefficient matrices of the auto-regressive process are determined by solving Eq. (11) and assigning α = 0.02 and c = 1000m/sec for Eq. (31). The results shown in Fig. 3 are in good agreement with the sample motions shown in Fig. 1.

When the stochastic characteristics, *i.e.*, the cross-correlation function of the field, and earthquake motions at several points in Fig. 1 are given up to time $t-1$ as observed motions, the motions at the intermediate points at time t are interpolated from Eq. (21). The results shown in Fig. 4(a) agree well with the sample motions shown in Fig. 1. Figure 4(b) is the interpolated earthquake ground motions for the case of α = 0.1 and c = 1500m/s. As the value of α increases the distortion of propagating wave becomes large. Even such a case the developed interpolation algorithm works well.

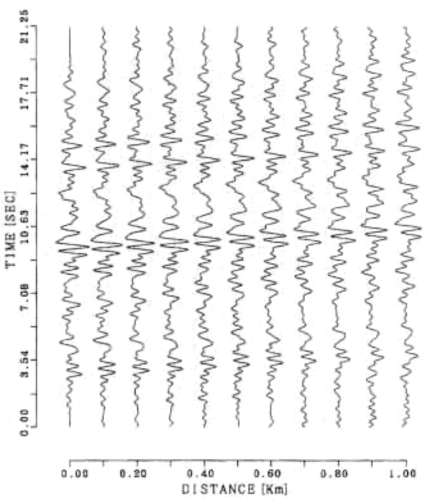
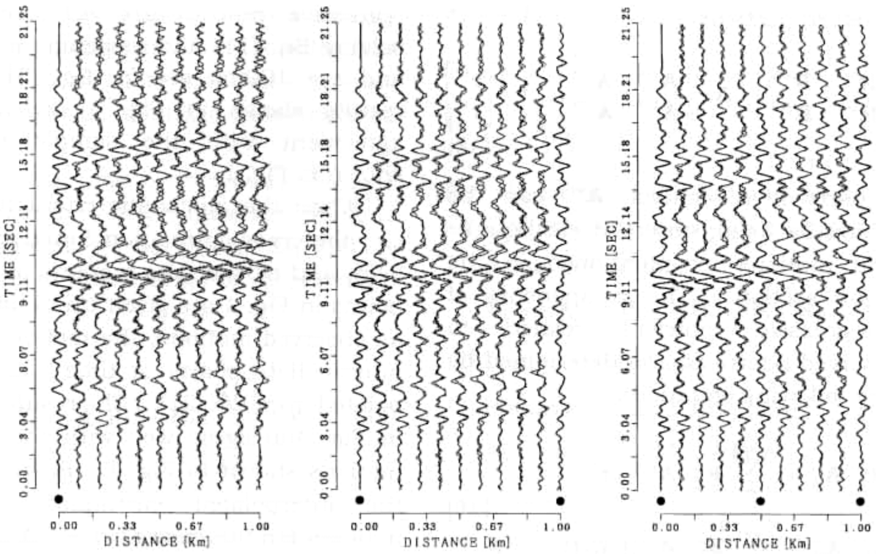
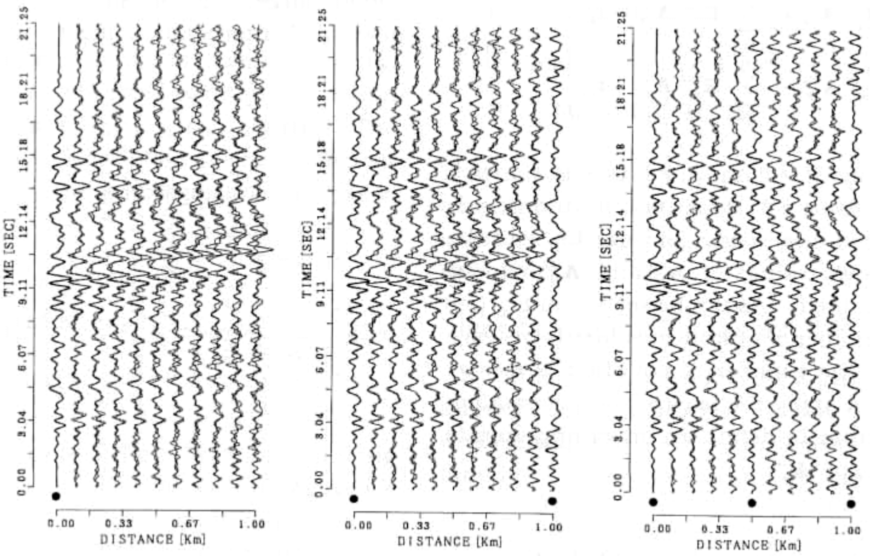


Fig. 3 Earthquake ground motions simulated by the autoregressive process defined by Eq. (4) with no system noise. The coefficient matrix of AR process are obtained by solving Eq. (11) provided the cross-correlation function of the concerned field is given by Eq. (31)



(a) the case of c = 1000m/s and $\alpha = 0.02 \times 2\pi$



(b) the case of c = 1500m/s and $\alpha = 0.1 \times 2\pi$

Fig. 4 Interpolated earthquake ground motions (thin lines) when earthquake motions (thick lines) at points expressed by • are given. For the comparison the sample earthquake motions simulated by Eq. (25) are shown by broken lines.

Increasing the number of observation points improves the accuracy of the posterior estimation of earthquake motion at an unobserved point. Consider a realistic two-dimensional field, in which an unobserved point is located at the center of the field and one observation point (No. 4) is added to the already distributed three observation points (No. 1, 2, 3) as shown in Fig. 5. The side length of each grid is 100m. Earthquake motion propagating in the 45° direction to horizontal axis with a wave velocity of 1000m/sec and α = 0.02 is assumed, and sample motions are simulated at five points including unobserved point (Fig. 5). The resulting time history for the unobserved point becomes the exact time history when the posterior estimation of earthquake motion is introduced. The interpolated motion at the unobserved point obtained from records providing three observation points is shown in Fig. 6(a) and the time history that takes into account the record for a newly added observation point is in Fig. 6(b). As the number of observation points increases, the estimated earthquake motion comes close to the exact time history as can be seen in Fig. 6(c).

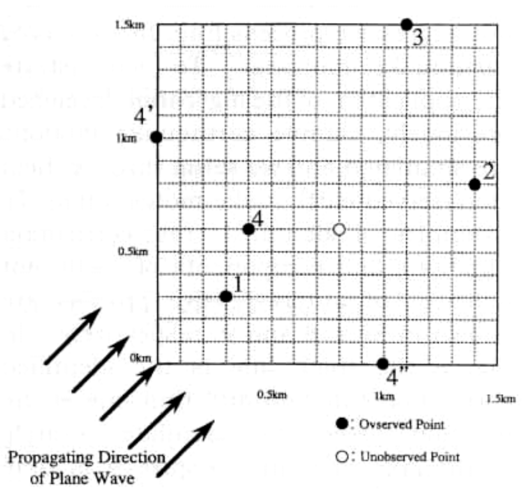
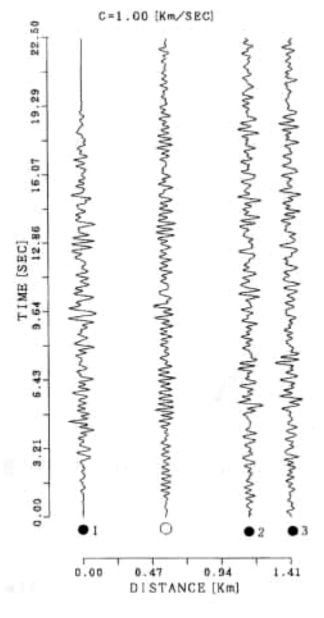
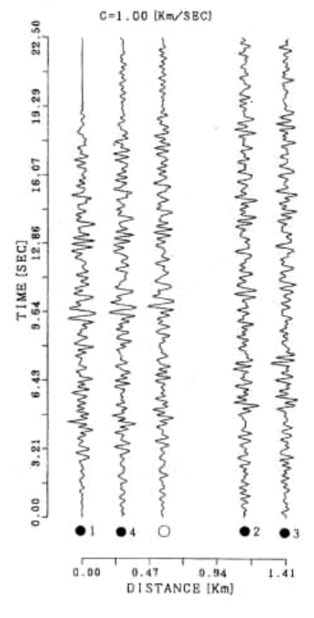


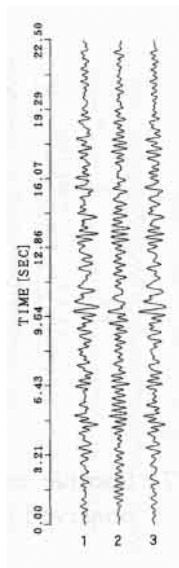
Fig. 5 Location of observation points and an unobserved point



(a) the case of three points observations (●)



(b) the case of four points observations (●)



(c) comparison between the exact ground motion (1) and interpolated ones (2: from case (a), 3: from case (b))

Fig. 6 Increase in interpolation accuracy of earthquake ground motion at an unobserved point as the number of observation points increases

When the stochastic characteristics of the field are not given apriori we must identify the cross-correlation function and the coefficient matrices of the autoregressive process from the observed earthquake motions. To demonstrate the application of the algorithm developed previously, sample earthquake motions are simulated and we select three of them as observed earthquake motions (Fig. 7). Examples of identified cross-correlation functions and components of coefficient matrices of autoregressive process are shown in Figs. 8 and 9, respectively. In Fig. 8 the thick line is the identified correlation function and thin line is the function used to simulate sample earthquake motions. Figure 9 is time histories of the identified autoregressive coefficient matrix components related to observed points. For conditional interpolation of earthquake ground motions the components of the coefficient matrices of autoregressive process related to unobserved points must be calculated by Eqs. (39) and (40). The accuracy of calculated results is shown in Fig. 10 in which the abscissa is the exact values and the ordinate the calculated values. Because all calculated components lie on the line with 45° to abscissa, we can confirm the algorithm to identify the autoregressive process only from observed data. Once the coefficient matrices of autoregressive process are identified we can interpolate earthquake motions at unobserved points from observed motions. Figure 11 shows the ability of the proposed method.

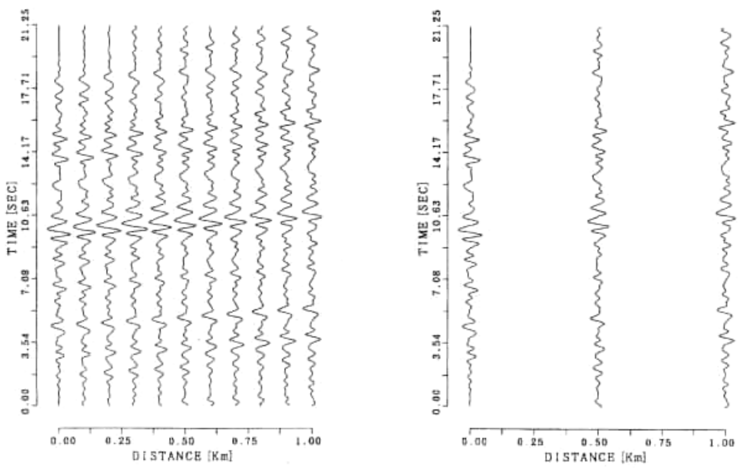


Fig. 7 Sample earthquake motions (left) and earthquake motions assumed to be observed (right)

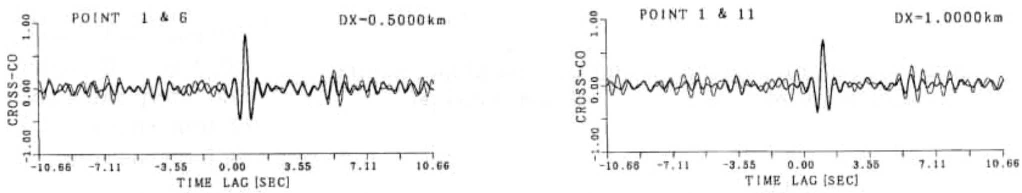


Fig. 8 Examples of identified cross-correlation function (thick line) from ground motions and one used to simulate sample earthquake motions (thin line)

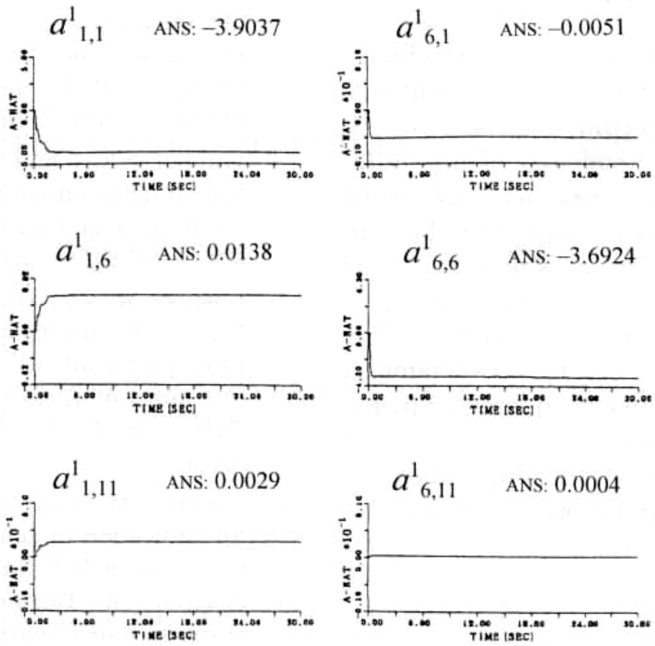


Fig. 9 Examples of identifying process of AR coefficient matrix components

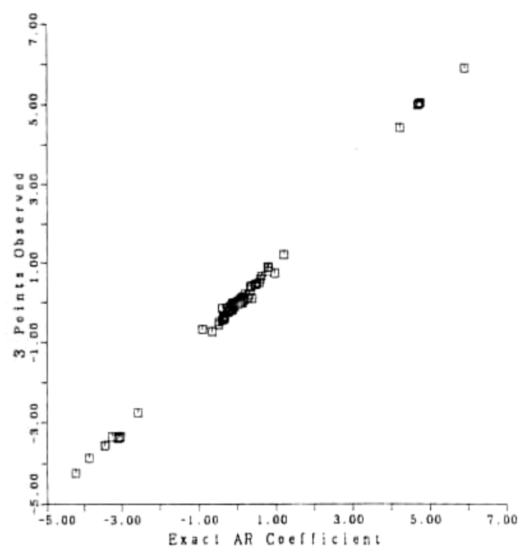


Fig. 10 Comparison of the identified AR coefficient matrix components with the exact ones

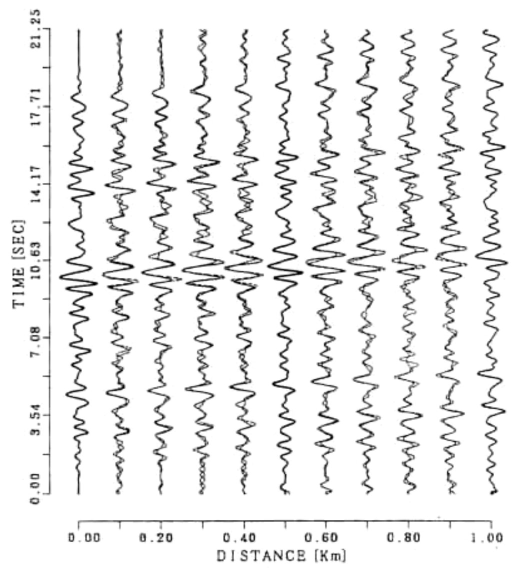


Fig. 11 Interpolated earthquake ground motions (thin line) by using identified AR matrix components when earthquake motions are assumed to be given at three points expressed by •

CONCLUSION

We have proposed a stochastic interpolation method for the conditional simulation of earthquake motion. It combines the methodology of the multi-autoregressive process for simulating earthquake motions and the Kalman filtering technique in which the state vector, including unknown parameter, is updated based on observations. This method provides the best estimator of earthquake motions at unobserved points that coincide with sample motions at observed points. Its application to earthquake ground motion simulation is illustrated by several examples.

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