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# A Simple Seismic Design Strategy Based on Displacement and Ductility Compatibility

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## ABSTRACT

Elementary but largely forgotten principles, relevant to the seismic behavior of structural systems comprising elements with very different characteristics, are reviewed. In such structures the system displacement at the ultimate limit state may be associated with very different displacement ductility demands imposed on its elements. It is shown how, by simulating nonlinear ductile structural response with bilinear modeling, the unavoidable restrictions on the system ductility capacity, necessary to protect elements with the smallest displacement potential, can be readily determined. Systems subjected to uniform and non-uniform translations at the ultimate limit state are considered. The latter are associated with torsional phenomena. The behavior and subsequent evaluation of distinct torsional mechanisms is presented. A seismic design strategy, very different from that embodied in current building codes, is postulated. The highlights of this strategy, strongly design rather than analysis oriented and claimed to be rational and extremely simple, are summarized.

## INTRODUCTION

Recent studies and reviews of established practices in structural seismic design revealed unintentional misuse of fundamental principles. In certain cases this may seriously alter the expected performance of structures designed for fully or limited ductile response. Mixed structural systems are particularly affected. Typical structures of this type are, for example, those where lateral force resistance is assigned to a set of reinforced concrete cantilever walls with markedly different dimensions and cross sections. Dual systems, in which ductile interacting cantilever structural walls and frames resist lateral forces, belong also to this group.

Mixed systems considered here are those which comprise vertical lateral force-resisting elements with markedly different elasto-plastic force-displacement characteristics. Because the aim of this review is only to highlight very fundamental, yet simple, principles, only single degree of freedom systems are considered.

In the process of developing diagnostic procedures for a quantitative evaluation of buildings with potential seismic risk in New Zealand, it emerged that existing codified procedures [1], addressing the torsional response of buildings, may not be suitable for this purpose. Therefore, a study, based on a return to fundamental principles, was undertaken to trace features that are more relevant to ultimate limit state design criteria. This indicated that attention should be focused on the identification of potential plastic mechanisms and associated displacement demands and capacities corresponding with ductile response of the constituent elements of structural systems used in buildings. It became evident that in this context strength rather than stiffness eccentricity is the principal parameter to be accounted for.

Results of these study, which emerged gradually, were published as they came to hand. Assumptions based on traditional concepts and extensively used in numerous relevant studies [2] were also adopted at the initial stages of this study [3,4]. With the recognition of more appropriate definitions of the transition of systems from the elastic to the plastic domain of response [5], these assumptions were subsequently revised. The summary of the findings of this study, several details of which appeared in various publications of the author, are presented here, particularly for the benefit of potential interested readers who may not be familiar with seismic design developments in remote New Zealand.

Emphasis is placed on recognition of structural behavior rather than on sophistication in analytical predictions. Therefore, the presentation is design rather than analysis oriented. This approach was strongly influenced by the introduction some 20 years ago, and used since, of the philosophy of "capacity design" in New Zealand, whereby the designer is invited to "tell the structure what it should do in the event of a major earthquake". Therefore, primarily ductile response, involving significant demands on differing elements by earthquake-imposed inelastic deformations, is addressed. The tools employed are extremely simple. However, some designers may consider them unconventional. This necessitates some readjustment of traditional concepts and definitions associated with elastic structural behavior. The strategy proposed assumes a good understanding of and feeling for structural behavior. This includes the simple laws relevant to kinematically admissible plastic mechanisms. Although explicit equations to quantify various features of behavior are presented, it is emphasized that a simple step-by-step process, each identifying a particular aspect of behavior, can be used to arrive at solutions without reliance on specific rules. When these solutions are not considered to be acceptable, simple changes of properties, particularly those relevant to element strength, can be introduced in order to arrive at a better or even optimal solution.

Two distinct mechanisms, one torsionally restrained, the other torsionally unrestrained, are considered in detail. Quantifying the response of these systems takes up a major part of the presentation. A brief introduction to the treatment of inelastic displacements, which may be imposed by earthquakes in any direction, is included.

The primary purpose of this study is to address performance criteria of ductile systems conforming to ultimate limit state requirements. These are:

- Earthquake-induced deformations, including those associated with system twist, should limit the expected ductility demand on any element to its quantified ductility capacity, $\mu_{\Delta i\,\max}$.

- Magnitudes of ultimate inter-story displacements, to be expected at locations remote from the center of twist, should not exceed those considered acceptable for ductile systems, typically 2.0 to 2.5% of the story height.

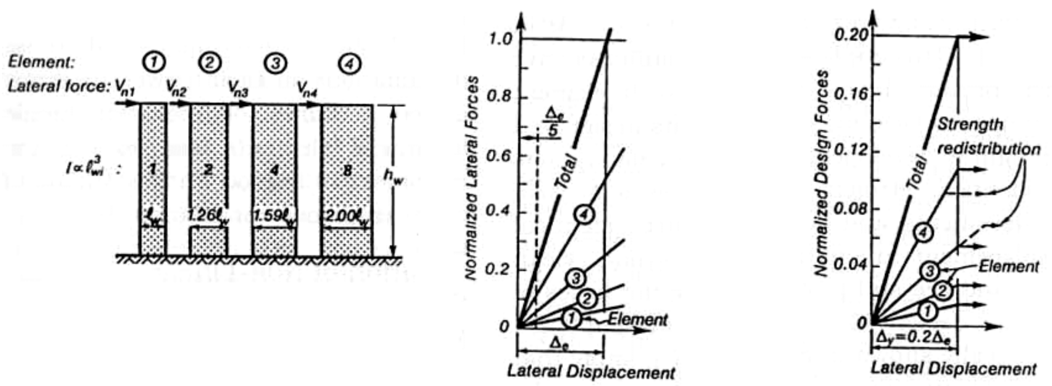
The purpose of considering torsion in ductile systems should be, therefore, to account for twist-imposed displacements on certain elements, rather than to provide torsional resistance. The word "twist" is used deliberately in this context to emphasis the need to address torsion-induced displacement demands.

The proposed strategy is equally applicable to force- and displacement-based seismic design approaches.

## THE THEORY OF ELASTICITY AND ITS TRADITIONAL INTERPRETATION

The requirements of deformation compatibility in statically indeterminate elastic structures are well established. Accordingly applied lateral forces are assigned to resisting elements in proportion of element stiffness. For example for prismatic cantilever elements of identical heights, such as shown in Fig. 1(a), the force, $V_i$, will generate a force in an element proportional to its flexural rigidity, *EI*. This principle stems form the requirement that deflections and curvatures of such elements, interconnected by infinitely rigid diaphragms, must be identical at all levels.

The familiar relationships used in assigning design strength to elements are illustrated for a specific example structure shown in Fig. 1(a). Four rectangular reinforced concrete walls with identical width are considered. The normalized second moments of area of the walls are 1, 2, 4 and 8, respectively, as shown. These relative values are not




(a) Interconnected elements (b) Elastic response (c) Elastic response based on traditional assumptions

Fig. 1 Wall element response based on the theory of elasticity affected by the allowance which may have been made for the reduction of element stiffness due to cracking of the concrete as long as the same proportion of reduction, as in this example, is applied to all four elements.

The corresponding share of each wall in resisting a unit base shear is shown in Fig. 1(b). The stiffness of the mixed structure ($K_s$ = 15 $k_1$) is then used to determine the fundamental period, $T$, of the system and hence the base shear corresponding with elastic response and consequent deflection, $\Delta_e$.

The design forces for the ultimate limit state of the structure, expected to behave in a ductile manner, are then reduced primarily as a function of the displacement ductility factor, $\mu_\Delta$ relevant to the system. The question must then arise: "What is the displacement ductility capacity of a mixed structural system?" In this example it is assumed [5] that $\mu_\Delta$ = 5. It is therefore implied that for typical cases where the "equal displacement concept" is deemed to apply, the yield displacement of the system is for example of the order of $\Delta_y = \Delta_e / 5$.

The traditionally assumed idealized bilinear response of the elements and the system is shown to an enlarged scale in Fig. 1(c). This implies that stiffness have been preserved and that each element commences yielding when its designated strength is developed. The assumptions of identical element and system yield displacements and consequent identical displacement ductilities beyond yield, have and are still providing the bases of seismic design [2]. The assumptions, graphically shown in Fig. 1(c), imply that the yield curvatures of all elements at the base are also identical.

In recognition of assured ductile response, a departure within certain limits, typically 30%, from strength allocation corresponding to elastic behavior, has been extensively used in some countries [6,7]. This implied that elements for which design forces derived from the analysis of the elastic structures have been reduced, would yield earlier than those elements to which design forces have been added in the process of strength redistribution. The dashed lines in Fig. 1(c) illustrate the presumed behavior resulting from redistribution of strength from element (4) to element (3).

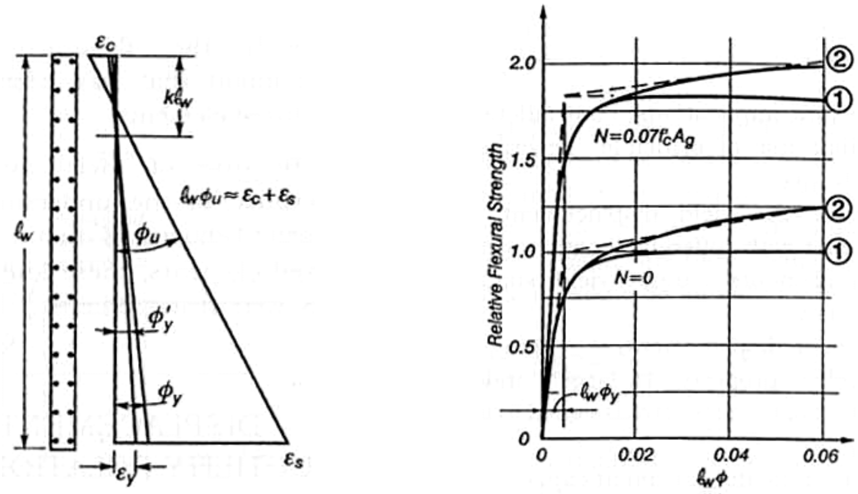
Fallacies relevant to the above widely used procedure are discussed in the next section.

## THE TRANSITION FROM THE ELASTIC TO THE PLASTIC DOMAIN OF BEHAVIOR

The predictions of the theory of elasticity are reliable. So are those of the theory of plasticity. However, many designers encountered difficulties when attempting to quantify the transition from one system to the other. With the use of bilinear behavior models, this nonlinear transition can be approximated. The precision of such models is considered to be adequate for the purposes of seismic design. The major appeal of these approximations is their power to throw light on features of realistic ductile response of elements and to aid the development of a good understanding of ultimate displacement relationships.

### Simulation of Non-Linear Moment-Curvature Relationships

The results of moment curvature analysis of typical rectangular sections of reinforced concrete walls are reproduced in Fig. 2. In fully cracked reinforced concrete section with uniformly




(a) Strain patters (b) Strength-curvature relationship

Fig. 2 Flexural strength-curvature relationships for typical rectangular wall sections

distributed reinforcement, non-linearity begins at the onset of yielding of the bars at the extreme tension fiber. The yield curvature at this stage is $\phi'_y$. This is shown in Fig. 2(a). It is convenient and adequate to simulate the subsequent nonlinear response with a bilinear relationship. The yield curvature, relevant to the idealized bilinear section response, termed here as the "reference yield curvature", $\phi_y$, can then be defined by extrapolation to a value which corresponds to the nominal or ideal flexural strength of the section. It has been shown [8] that the value of $\phi_y$ is relatively insensitive to the amount of reinforcement used and the axial compression load intensity, *N*, normally encountered in walls of multistory buildings. As the strain patterns in Fig. 2(a) show, this reference curvature is simply:

$$\phi_y = \frac{\lambda \epsilon_y}{\ell_w} \tag{1}$$

where $\ell_w$ is the length of the wall and $\lambda$ is a constant quantifying the influence of the ratio of $\phi_y / \phi'_y$ and the depth of the neutral axis at the onset of yielding [8]. For an identical pattern of forces applied to walls, as in Fig. 1(a), it is evident that the corresponding reference yield displacement of an element, $\Delta_{yi}$, will be proportional to its reference yield curvature, $\phi_{yi}$, at the base. Figure 2(b) shows how post-yield stiffness, represented by curves (2), can be adequately simulated by a linear relationship.

## Implications of the Redefinition of Yield Displacements

From the simple relationship illustrated in Fig. 2 an important conclusion, that has not been widely recognized in the past, must be drawn, i.e., yield deformations, such as $\phi_{yi}$ and $\Delta_{yi}$ of cantilever elements, are inversely proportional to the length of the elements [5,8]. Yield deformations, do not bear any relationship to flexural rigidity, *EI*! The reference yield displacement of an element, $\Delta_{yi}$, can then be used to quantify its ultimate or acceptable displacement capacity, $\Delta_{ui}$, by means of the displacement ductility ratio

$$\mu_{\Delta i,\,\max} = \frac{\Delta_{ui}}{\Delta_{yi}} \tag{2}$$

Important implications, contradicting the familiar use of principles described previously, are:

1. Because the yield displacement of elements with different length varies, such elements cannot yield simultaneously.
2. The yield displacement, $\Delta_{yi}$, being a geometric property, is largely independent of the strength assigned to an element.
3. The ultimate displacement capacity of a mixed system, $\Delta_u$, subjected to uniform translation, will thus be limited by that of the longest element, $\Delta_{ui}$, having the smallest yield displacement.
4. Because yield displacements are insensitive to the strength of elements, design strengths may be assigned within broad limits to elements in any arbitrary manner. Some techniques of strength distribution, distinctly different from that based or traditionally defined element stiffness, are particularly attractive [5].
5. Once the yield displacement is defined, by similarity to the relationships shown in Fig. 2(b), for purposes of seismic design, elements stiffness can then be conveniently quantified as:

$$k_i = \frac{V_{ni}}{\Delta_{yi}} \tag{3}$$

where $V_{ni}$ is the nominal strength of the element, as constructed. An important feature of this definition, to be noted, is that, contrary to the customary usage in accord with the theory of elasticity, stiffness is proportional to strength. Hence, the stiffness of any of the wall elements, shown in Fig. 1(a), with given dimensions will vary proportionally with the flexural reinforcement content that has been provided in these elements.

Because of their importance and relevance to the understanding of the seismic behavior of ductile systems with mixed elements, these known principles [5,8] were restated here.

## DISPLACEMENT AND DUCTILITY RELATIONSHIPS IN MIXED STRUCTURAL SYSTEMS SUBJECTED TO UNIFORM TRANSLATIONS

Once the idealized bilinear force-displacement relationship for each element of a mixed system is quantified, a similar relationship for the system as a whole can be readily established. This then can be utilized to satisfy design criteria for the complete ductile systems.

The principles relevant to the system shown in Fig. 1(a) are examined further. It is assumed, as an example, that strengths have been assigned to elements in accordance with traditional procedures of elasticity, as shown in Fig. 1(c). The modeling of force-displacement relationships, based on the previous definition of yield displacements, is presented in Fig. 3. It will be seen that for the estimation of the displacement ductility capacity of a mixed systems, this being a ratio, it is sufficient to use relative or normalized values of strength and yield displacements.

The relative yield displacements of elements in Fig. 1(a) are taken as $\Delta_{yi} = 1/\ell_{wi}$. For example $\Delta_{y1} = 1/1 = 1.0$ and $\Delta_{y8} = 1/2 = 0.5$. The force-displacement relations for the elements, shown in Fig. 3,

$k_1 = 1.00 \times 0.067 = 0.067$
$k_2 = 1.26 \times 0.133 = 0.168$
$k_3 = 1.59 \times 0.267 = 0.425$
$k_4 = 2.00 \times 0.533 = 1.067$

$\Sigma k_i = K_s = 1.727$

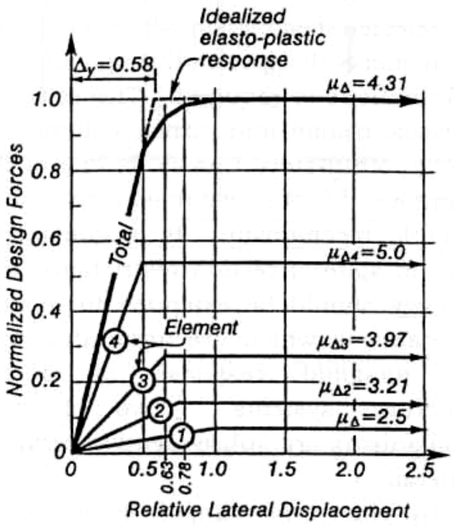



Fig. 3 The modeling of the elasto-plastic behavior of different elements

are thus readily defined. The element forces shown and established in Fig. 1(c) are normalized in terms of the total design base shear required for the ductile system. The relative stiffness of each element is thus $k_i = V_{ni} / \Delta_{yi} \propto \ell_{wi} V_{ni}$ (Eq. (3)). Numerical evaluation of these is shown alongside Fig. 3.

The total multi-linear response, shown in Fig. 3, can again be simulated by a bilinear relationship shown there by the dashed lines. The reference yield displacement of the system, comprising the four walls given in Fig. 1(a), is thus

$$\Delta_y = \frac{\Sigma V_{ni}}{\Sigma k_i} = \frac{1.000}{1.727} = 0.58 \qquad (4)$$

The maximum acceptable system displacement demand must be limited to the displacement capacity of the most critical element. In this example it is assumed that all elements are intended to be detailed to have a displacement ductility capacity of $\mu_{\Delta i,\max} = 5.0$. Therefore, the critical ultimate displacement is that of the element with the smallest yield displacement, i.e., $\Delta_{u4} = 5 \times 0.5 = 2.5$. Therefore, the displacement ductility demand on the system should be limted to:

$$\mu_\Delta = \frac{\mu_{\Delta i,\max}\,\Delta_{yi,\min}}{\Delta_y} = \frac{2.5}{0.58} = 4.31 \qquad (5)$$

The ductilities relevant to the non-critical elements are also shown in Fig. 3.

This example shows that the original design assumptions may need to be reviewed to determine the design base shear that corresponds with the period, $T$, of the system and the system stiffness of $K_s = 1.727$, as well as the displacement ductility capacity of the order of 4.3. Figure 3 shows that element (1), in spite of being part of a fully ductile system, could be detailed to satisfy requirements for elements with limited ductility.

The relationships presented in Fig. 3 show that:

1. Significant differences in displacement and ductility estimated arise when the two approaches previously described are employed. A comparison of Fig. 1(c) and Fig. 3 demonstrates this.
2. The displacement ductility capacity of a mixed system is governed by that of the element of greatest length, irrespective of its share in the total base shear resistance!
3. Element stiffnesses are affected by the strength assigned to them. Hence the larger the base shear resistance of the longest element, the smaller the system yield displacement, resulting in decreased system displacement capacity.

4. The designer has great freedom in assigning strength to elements, without having to observe customary limitations on the redistribution of element resistances. However, such a choice will affect the strength-dependent element stiffnesses and hence to a small degree the system stiffness. The latter controls the system yield displacement and hence the reduction of the system displacement ductility capacity in relation to the ductility capacity of the critical element of the mixed system.

## DISPLACEMENT AND DUCTILITY RELATIONSHIPS IN MIXED STRUCTURAL SYSTEMS SUBJECTED TO VARIABLE TRANSLATIONS

Because of the inevitable lack of symmetry, most ductile buildings will be subjected also to system rotations due to torsional effects. This means that ductile translatory elements, such as shown in Fig. 1(a), when not located in the same plane, may be subjected to different displacements when the ultimate limit state is being approached. In this section the effects of the differences of element displacements on element and system displacement ductility demands are examined. The approach proposed [5] is radically different from those embodied in seismic codes [1]. Therefore, no reference to the latter procedures, not considered to be rational with respect to ductile response, will be made.

### The Need to Establish Mechanisms

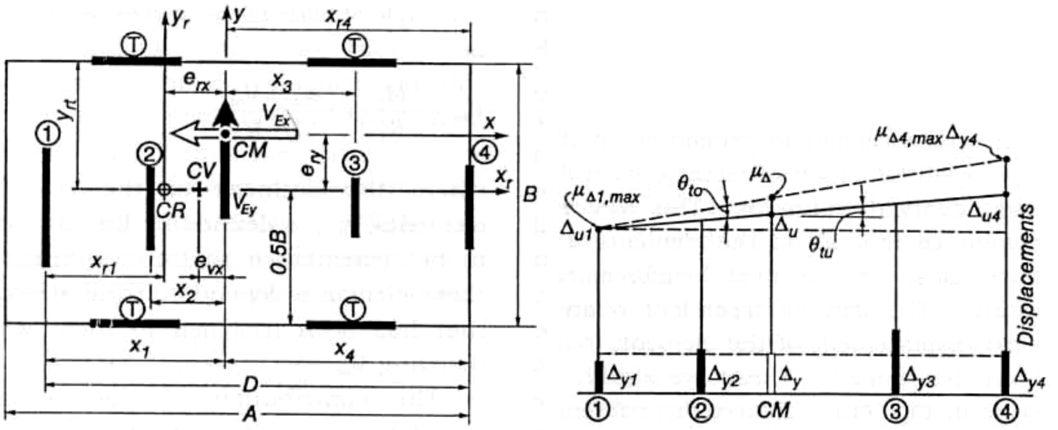
In accordance with the philosophy of capacity design [7,9], the choice of a suitable and kinematically admissible plastic mechanism is a prerequisite for the successful seismic design of ductile systems. By enforcing a definitive strength hierarchy of elements, it is then possible to ensure that, irrespective of the characteristics of ground motions, only the selected mechanism will be mobilized when energy dissipation through inelastic deformations is required. The selection of weak beams and strong columns in ductile multi-story frames, is one of the examples of the establishment of a suitable mechanism. It is considered that, in spite of certain restrictions, this strategy should be extended to include mechanisms which are associated with the torsional response of ductile structural systems. Two of such mechanisms are subsequently examined in detail.

To this end simplified structural models with distinct lateral force-resisting elements, widely used in similar studies and subsequently illustrated, will be used. The elements shown imply the use of structural walls, to which specific references have already been made. This modeling is used only to illustrate, in the simplest way, relevant principles. The elements so shown may represent also ductile frames, for which associated properties can also be readily determined.

### Torsionally Restrained Systems

The simplified model of a commonly encountered structural system is shown in Fig. 4. When, as a result of an only applied base shear, $V_{Ey}$, the ductile response of the system is to be considered, the most important information required will be the center of resistance, *CV*, of the four translatory elements, (1), (2), (3) and (4). This can be readily derived from

$$e_{vx} = \frac{\Sigma\, x_i\, V_{ni}}{\Sigma\, V_{ni}} \qquad (6)$$



(a) Plan (b) Displacement patterns

Fig. 4 A torsionally restrained example structure

where $V_{ni}$ is either the nominal strength of the element, as constructed, or the maximum lateral force developed in the element when, for reasons to be explained subsequently, it responds within the elastic domain, and $x_i$ is the distance of the element measured from the center of mass, subsequently referred to only as *CM*. Similar considerations enable *CV*, based on the nominal strength of the (T) type elements, utilized under the action of $V_{Ex}$, to be located. The strength eccentricity, defined by Eq. (6), locates thus the position with respect to *CM* of the resultant of the sum of the forces developed in the four ductile translatory elements when the ultimate limit state in the *y* direction is being approached.

It is evident that when ductility, associated with inelastic displacements in the *y* direction only, is to be developed, a torsional moment needs to be resisted. This is uniquely defined by the strength eccentricity, $e_{vx}$. The torsion-induced forces will be resisted only by the (T) type transverse elements shown in Fig. 4. Therefore, the resulting torsional rotation of the rigid diaphragm, i.e., the angle of twist, $\theta_{tu}$, can be determined, provided that the transverse elements respond in the elastic domain. This condition should be the aim of the designer.

When the transverse elements remain elastic, while the translatory elements are subjected to variable displacement ductility demands, consideration of statics suggest that the angular rotation of the system is controlled, and remains independent of the inelastic translatory displacements that may be imposed by an earthquake. It is for this reason that such systems are defined here as being torsionally restrained. It may be possible, however, that the rotating inertia of a distributed mass may introduce additional torsion.

*The properties of an example structure*

Figure 4(a) shows the arrangement of lateral force-resisting elements in a typical torsionally restrained system. To illustrate features of ductile behavior, the variable inelastic displacements of elements (1) to (4) under the static action of a unit base shear, $V_{Ey}$, will be considered. It has been stated that strength to elements can be assigned rather arbitrarily. In this example with cantilever structural walls, strength required to resist the unit base shear, $V_{Ey}$, has been assigned in proportion to the square of the length of rectangular walls with identical width [5]. This has the advantage that all of the elements will have close to identical reinforcement ratios. The lengths-dependent relative yield displacement of the elements have been determined. These are shown to scale in Fig. 4(b). Hence, the reference yield displacement of the system in the $y$ direction, $\Delta_y$, can be determined from Eq. (4). This displacement at *CM* is also shown in Fig. 4(b). It is evident that under uniform translation of the diaphragm, element (1) will be the first one to yield while element (3) will be the last one.

With the knowledge of element strength, the center of the strength, *CV*, is readily found from Eq. (6). Note that the allocation of strength to elements of the system was made with disregard to any eccentricity that may arise. With *CM* being given, the strength eccentricity, $e_{vx}$, is thus established. The coordinates, $x_i$, with respect to *CM* of all elements are also shown in Fig. 4(a).

*The prediction of the angle of twist*

The resulting torque, $M_t = e_{vx} \Sigma V_{ni}$, is resisted by the elastic (T) type elements. The primary role of these is, however, to resist seismic forces acting in the $x$ direction. Hence, their strength will be of the order of 0.25 $V_{Ey}$. When the torsion-induced forces are compared with the strength of these elements, it will be immediately evident whether they will respond within the elastic domain, an assumption which will be used in the subsequent study of torsional behavior. Therefore, using first principles for deriving the relevant torsional stiffness, $K_t$, the angle of twist of the system is found to be

$$\theta_{tu} = \frac{M_t}{K_t} = \frac{e_{vx} \, \Sigma V_{ni}}{\Sigma y_i^2 \, k_{xi}} \qquad (7)$$

where the sitffness of the (T) type elements, $k_{xi}$, is defined by Eq. (3). It is to be remembered that the stiffness of these elements depends on the strength that has been assigned to them when resisting $V_{ex}$.

The contribution of the inelastic translatory elements (1) to (4) to torsional strength or to torsional stiffness is negligible!

*The estimation of element displacement and system displacement ductility capacities*

It is evident that, unless the angle of twist is large, the critical displacement demand will be that on the element with the greatest length, i.e., the smallest yield displacement. In the example structure shown in Fig. 4(a) this is element (1). Therefore, with the knowledge of the angle of twist at the ultimate limit state, $\theta_{tu}$, and the maximum acceptable displacement that may be imposed on element (1), $\Delta_{u1} = \mu_{\Delta 1,\max} \, \Delta_{y1}$, the corresponding displacement pattern of the system, as shown in Fig. 4(b), can be established. This then indicates the maximum acceptable displacement at *CM*, $\Delta_u$. From the geometry of the displacement profile, shown by a full line, it may then be readily shown that the system displacement ductility demand must be limited to

$$\mu_\Delta = \frac{\Delta_u}{\Delta_y} = ( \mu_{\Delta 1,\max} \, \Delta_{y1} + x_1 \, \theta_{tu} ) \, / \, \Delta_y \qquad (8)$$

if the displacement ductility capacity of element (1) is not to be exceeded. Distances, such as $x_1$ are to be taken from the center of the coordinate system at *CM*, and anti-clockwise system rotation, as in Fig. 4(b), is taken as negative.

The optimal solution for the system displacement ductility capacity in this case is obtained when the displacement ductility capacities of the edge elements, $\mu_{\Delta i,\max}$, not necessarily identical, are attained simultaneously. With the notation used in Fig. 4, this situation is associated with the optimal angle of twist

$$\theta_{to} = \frac{\mu_{\Delta 1,\max}\ \Delta_{y1} - \mu_{\Delta 4,\max}\ \Delta_{y4}}{D} \qquad (9)$$

This corresponds to a strength eccentricity of

$$e_{vxo} = \frac{K_t\ \theta_{to}}{\Sigma V_{ni}} \qquad (10)$$

where $K_t$ is the torsional stiffness provided by the elastic transverse (T) type elements in accordance with Eq. (7). When the angle of twist, $\theta_{tu}$, is found to be less than the optimal value, given by Eq. (9) , Eq. (8) will limit the acceptable displacement ductility demand on the system. However, when $\theta_{tu} > \theta_{to}$, the system displacement ductility demand needs to be limited to

$$\mu_\Delta = (\ \mu_{\Delta 4,\max}\ \Delta_{y4} - x_4\theta_{tu}\ )\ /\ \Delta_y \qquad (11)$$

because element (4) is the controlling system displacement, $\Delta_u$.

The displacement patterns in Fig. 4(b) show that, contrary to common design assumption, the optimal torsional response of systems with different lateral force-resisting elements, is not associated with zero strength eccentricity. The elimination of strength eccentricity is only desirable when all elements have the same yield displacement. The designer could, if desired, redistribute strength from one element to another one to approach the optimal mechanism implied by Eq. (9).

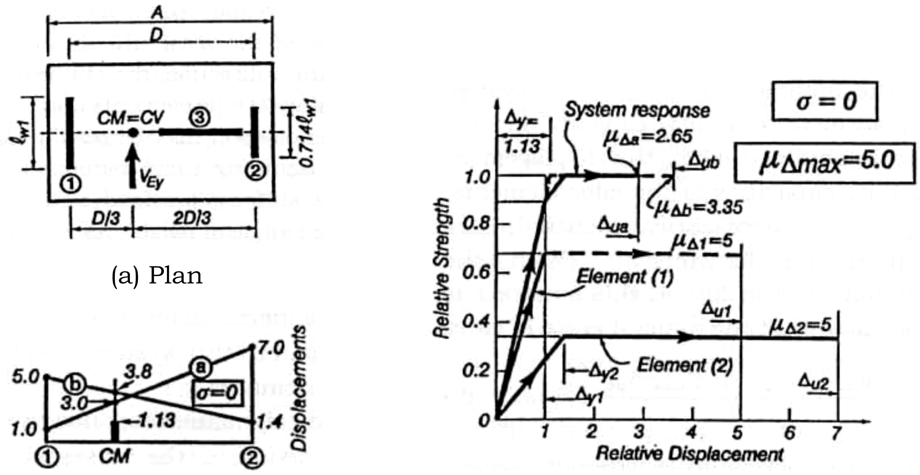
As a result of inelastic seismic response of the structure shown in Fig. 4(a) in the *x* direction, the stiffness of the restraining (T) type elements may degrade. The phenomenon may be readily allowed for by assuming some reduction of the torsional stiffness provided, i.e., using an increased angle of twist. As was pointed out, a degradation of torsional stiffness does not necessarily result in further restriction on the system displacement ductility demand.

During dynamic response the torque-induced twist of the system will also mobilize the rotary inertia of the mass, a feature which is not included in the above considerations. It is likely that a dynamic torque, additional to that due to the strength eccentricity, will be generated. Thereby the angle of twist at critical instants of the response will increase. The phenomenon needs further study.

## Torsionally Unrestrained Systems

Figure 5(a) shows the model of a structure which will be referred to as being torsionally unrestrained. Corresponding elastic structures have been extensively studied. Therefore, the following discussion addresses features of ductile behavior only.

As Fig. 5(a) implies, once at least one of the two elements has yielded, no static torque can be resisted. Hence, at this stage no strength eccentricity with respect to the total lateral force, $V_{Ey}$, can exist. Therefore, the center of resistance of the system, *CV*, must necessarily coincide with *CM*. If the nominal or ideal strength of elements, $V_{ni}$, happens to be exactly as required by equilibrium requirements, both elements can be expected to yield simultaneously. However, if the nominal strength of one of the elements is in excess of that required,




(a) Plan

(b) Displacement patterns (c) Displacements and ductilities

Fig. 5 The response of a torsionally unrestrained system

that element may not yield. In this case the inelastic displacement of the system, measured at *CM*, will depend entirely on the ductility demands imposed on the yielding element. It is for these reasons that these systems, not being able to offer any restraint with respect to torsional rotations, i.e., twist at the ultimate limit state, is best described as being torsionally unrestrained.

It may be noted that any torque resulting from eccentricity of the base shear force at right angle to $V_{Ey}$ in Fig. 5(a), can be readily resisted by elements (1) and (2), provided they respond within the elastic domain.

These elementary principles are recapitulated here only, because existing code provisions [1] do not address this type of structure. Therefore, the ductility capacity of such systems, as will be shown subsequently, may be underestimated.

*(1) Limitations on the system displacement ductility capacity*

The yield displacement of the system, i.e., that of *CM*, associated with the simultaneous development of the yield displacements of both elements, is

$$\Delta_y = \beta\, \Delta_{y1} + \alpha\, \Delta_{y2} \tag{12}$$

For the specific example in Fig. 5(a) $\alpha = 1/3$ and $\beta = 2/3$.

Because the yield displacements of elements (1) and (2) are different, at this stage the system will rotate without the development of any torque.

Although the designer may have assigned to elements (1) and (2) in Fig. 5(a) strength according to the laws of statics, i.e., $V_1 / V_2 = \beta / \alpha = 2$, the nominal strength of the elements, as constructed, is likely to be different, so that $V_{n1} / V_{n2} \neq \beta / \alpha = 2$. Therefore, it should be assumed that under static actions simultaneous yielding of the two elements will not occur. To establish the relevant relationships, the effects of post-yield stiffness.

$$k_p = \sigma\, k_i \tag{13}$$

of elements may be considered, where $k_p$ is the post-yield stiffness, this being a fraction, $\sigma$, of the initial stiffness, $k_i$, defined in Fig. 5(c) and by Eq. (3).

When element response is perfectly elasto-plastic, i.e., $\sigma = 0$, as implied in Fig. 5(c), it must be assumed that one of the elements, shown in Fig. 5(a), will not yield. To establish the simple displacement relationships, that can be derived as part of routine design calculations, the specific example shown in Fig. 5(a) will be used. When yielding of element (2) commences at the attainment of its nominal strength, $V_{n2}$, the force generated in element (1) will be $V_1 = (\beta / \alpha) V_{n2} = 2 V_{n2}$. When this force is smaller than the nominal strength of element (1), $V_{n1}$, developed under the static force $V_{Ey}$, it cannot yield. The displacement of element is at this stage

$$\Delta_1 = \frac{V_1}{k_1} = \frac{\beta V_{n2}}{\alpha k_1} = \frac{2 V_{n2}}{k_1} \tag{14}$$

If the displacement ductility capacity of the yielding element (2), $\mu_{\Delta 2,max}$, is not to be exceeded, the simple geometry of the displacement pattern, marked (a) in Fig. 5(b), shows that the system displacement ductility demand should be limited to

$$\mu_\Delta = ( \beta \Delta_{y1} + \alpha \mu_{\Delta 2,\max} \Delta_{y2} ) / \Delta_y \tag{15}$$

With the introduction of a convenient geometric system parameter

$$\psi = \frac{\alpha \Delta_{y2}}{\beta \Delta_{y1}} = \frac{\alpha \ell_{w1}}{\beta \ell_{w2}} \tag{16}$$

Eq. (1) simplifies to

$$\mu_\Delta \le \frac{\psi \mu_{\Delta 2,\max} + 1}{1 + \psi} \tag{17a}$$

When it is expected that element (1) rather than element (2), seen in Fig. 5(a), will yield, it is found that to protect element (1) against excessive displacement demands, the system displacement ductility demand should be limited to

$$\mu_\Delta \le \frac{\mu_{\Delta 1,\max} + \psi}{1 + \psi} \tag{17b}$$

In the design of such torsionally unrestrained systems, the magnitude of the expected displacement ductility demand should be limited to the lesser of that given by Eq. (17a) or Eq. (17b).

For the specific example shown in Fig. 5(a), for which $\mu_{\Delta i,\max} = 5.0$, $\ell_{w1} / \ell_{w2} = 1.4$, $\alpha = 1/3$ and $\beta = 2/3$, so that $\psi = 0.7$, is found that from Eq. (17a) that $\mu_\Delta = 2.65$ and from Eq. (17b) that $\mu_\Delta = 3.35$. The associated displacement patterns are shown in Fig. 5(b).

Corresponding bilinear force-displacement relationships are seen in Fig. 5(c), where the full lines show the condition when element (1) does not yield, while the dashed lines depict the relationships when element (2) does not yield.

The mechanisms shown in Fig. 5 imply that by means of an accidental eccentricity, additional design strength assigned to the structure designed to current code prescriptions may not be fully utilized. The above approach, based on criteria of static equilibrium and the kinematics of a simple mechanism, suggests that current code requirements [1] may be unconservative for torsionally unrestrained systems.

*(2) The inevitable loss of torsional restraint*

Provided that the strength of the non-yielding element in Fig. 5(a) is only moderately in excess of that required by equilibrium criteria, this element may eventually also commence yielding when the yielding element has some post yield stiffness, i.e., where $\sigma > 0$. An evaluation of these cases, similar to those of Eqs. (17a) and (17b), has also been suggested [10], but could not be presented here.

As Fig. 5(b) demonstrates, the displacements at *CM*, and hence the system displacement ductility capacity, is significantly restricted if the displacement ductility capacity of the yielding element is not to be exceeded. Therefore, torsionally unrestrained systems should preferably be avoided. However, there are circumstances when torsional restraint may drastically degrade or disappear. Such cases have also been studied [10]. Only the highlights of typical situations can be briefly presented here.

A system, such as seen in Fig. 4(a), may appear to be torsionally restrained. However, the nominal strength of the transverse (T) type elements may be insufficient to resist the torsion-induced forces after the translatory elements have entered the inelastic domain. For example, this may be the case where the dimension *B* in Fig. 4(a) is much smaller than *D*. The torque, $M_{to}$, which can then be sustained depends on the nominal strength, $V_{nt}$, of the transverse elements. This situation will arise when the strength eccentricity, $e_{vx}$, would need to be larger than that associated with $M_{to}$. When the transverse elements at one or the other edge of the plan (Fig. 4(a)) have yielded, the base shear capacity of the system will reduce and one of the translatory edge elements will return to the elastic domain. The treatment of such a case simply follows the principles presented in section (1).

Situations need also to be considered when an earthquake imposes significant displacements in directions other than along the principal axes of the floor plans, i.e., the *x* and *y* directions in Fig. 4(a). In such cases ductility demands are to be expected simultaneously on all lateral force-resisting elements of the system. It is evident that when, as a result of the base shear $V_{Ex}$, the (T) type elements yield, they will no longer be capable to resist torsion-induced forces resulting from a strength eccentricity $e_{vx}$. In such cases strength eccentricities should vanish because static torsion of any kind cannot be resisted. The system will then degrade into two torsionally unrestrained mechanisms, associated with $V_{Ey}$, and $V_{Ex}$, respectively. While displacement ductility capacities in the principal *x* and *y* directions may be drastically reduced, the displacement capacity of the system in a skew direction may well be in excess of that imposed in that direction by the design earthquake. A quantitative evaluation of the relevant properties has been proposed [10].

*(3) Likely effects during dynamic response*

As stated earlier, in this study only static equilibrium and the kinematics of plastic mechanisms were considered. Subsequent studies of torsional mechanisms, with the mass concentrated at CM, showed excellent agreement with the postulated behavior. However, when a distributed mass over the floor area is considered, the rotary inertia of this mass may generate a torque in accordance with the principles of dynamic equilibrium. The magnitude of such a torque in a torsionally unrestrained system (Fig. 5(a)) is limited by the strength of one of the elements in excess of that predicted by static equilibrium criteria.

To illustrate this principle the specific example model structure, shown in Fig. 5(a) and discussed in the beginning of section (1), is re-examined. Let it be assumed that the ratio of the nominal strengths of the two elements is such that $V_{n1} / V_{n2} = 2.2 > \beta/\alpha = 2$. According to static equilibrium the 10% excess strength of element (1) cannot be utilized even when significant post-yield stiffness exists. However, during dynamic response due to the excess strength of element (1) a torque $M_{tdyn} = 0.1 \times 0.667 \times D/3 = 0.222\,D$ can develop and the unit base shear resistance of the system, utilizing the full strength of both elements, increases to 1.0667. The sense of the resulting twist is opposite to that shown by line (1) in Fig. 5(a). Thereby element (1) may also enter the inelastic domain of response. As a consequence the angle of twist and the displacement demand on element (2) will reduce for the same system displacement ductility demand at *CM*. It is thus seen that in this case the rotary moment of inertia of the mass opposes the imposed diaphragm rotation and thereby allows a larger system displacement ductility to be developed. Hence, for torsionally unrestrained systems Eq. (17) may be considered to furnish an upper bound solution.

## CONCLUSIONS

1. The assessment of the ductile seismic response of systems, particularly with mixed elements, requires a re-examination of widely used practices [11]. One feature is the transition of elements from the elastic to the plastic domain of behavior. A simple yet promising seismic design approach, involving inevitable but acceptable approximations, should utilize bilinear modeling of element behavior.
2. To enable the transition from elastic to plastic response to be quantified, a redefinition of some traditionally used terms, such as stiffness and yield displacement, is necessary. Instead of using the flexural rigidity, *EI*, the reference yield displacement of an element should be based on the reference yield curvature of the critical section of the plastic hinge, as constructed. This enables a realistic assessment of the acceptable displacement ductility demand on any element as constructed to be quantified.
3. Having defined the most importance of the characteristics of a lateral force-resisting element, i.e., its strength and yield displacement and hence stiffness, its acceptable inelastic deformation capacity is also uniquely defined.
4. The displacement of a mixed structural system at the ultimate limit state, when subjected to uniform translation, is controlled by the deformation capacity of the element with the smallest yield displacement.
5. A definition of the yield displacement of a mixed structural system enables the limitation on the displacement ductility demand on the system, as a function of the displacement ductility capacities of critical elements, to be readily established.
6. A rational approach to the seismic design of building structures, which are expected to response in a ductile manner, requires that uniquely definable plastic mechanisms can be mobilized. This principle is widely recognized and applied when the translatory response of frames or walls is considered [7,9]. No equivalent approach to the definition of mechanisms involving torsion, which is addressed in this paper, appears to have been formulated.
7. When torsion-induced displacements occur, the primary aim of the design strategy should be to ensure that the expected displacement demand on the system does not lead to demands that exceed the displacement ductility

capacity of elements, particularly those remote from the center of twist. Critical elements will, in general, be those with the smallest yield displacement.

8. The identification and understanding of plastic mechanisms, as affected by system twist, enables the acceptable system displacement ductility demand, as a function of the displacement ductility capacity of critically situated vertical lateral force-resisting elements, to be estimated when, as a result of torsional phenomena, element translations within the system vary.
9. Two fundamentally different torsional mechanisms are postulated. In one, much to be preferred, elastic transverse elements are assigned to resist torsion and to control system twist, while translatory elements are subjected to different inelastic displacements. This system is defined as being "torsionally restrained". In the other system, preferably to be avoided, elements capable of resisting torsion during inelastic translatory response, are absent or inadequate. As a result, one edge element may not enter the inelastic domain while the element at the opposite edge of the plan may be subjected to excessive ductility demands. This system is defined as being "torsionally unrestrained".
10. The identification of mechanisms which can be mobilized independently in each of the two principal orthogonal directions of the building, enables also the maximum potential inelastic displacement and base shear capacities, associated with skew earthquake attacks to be estimated. Torsionally restrained mechanisms subjected to inelastic skew displacement must be expected to degenerate into torsionally unrestrained mechanisms.
11. Code provisions, used the world over and based on the properties of elastic systems, do not appear to address the relevant vital features, that is, maximum element displacements generated during ductile seismic response. The key parameters of current code procedures are adjustable stiffness eccentricities, utilized to provide increased torsional resistance [1,2]. Thereby increased translatory design strength is achieved. Therefore, their intentions for ductile performance remain obscure. The important quantify relevant to ductile response is claimed here to be the strength rather than stiffness eccentricity. Only the former gauges realistically the torque to be sustained at the ultimate limit state.
12. The magnitude of the strength eccentricity is under the control of the designer. Without changing acceptable element dimensions, strength can be assigned to or redistributed between translatory elements so as to result in more favorable strength eccentricities. The strategy allows very large freedom for the designer to conceive and enforce torsioally well conditioned response.
13. A reduction or elimination of strength eccentricity in systems comprising elements with different yield displacements, is not likely to result in a more even distribution of element displacement ductility demands, a very desirable design aim.
14. The identification of torsional mechanisms can be of considerable benefit at the conceptual stage of the design. Moreover, it enables the potential displacement ductility

capacity of existing systems requiring a seismic review, and comprising elements with different identified strength and ductility properties, to be estimated. Established code provisions cannot be utilized for this purpose.

15. The design strategy postulated here was based on criteria of static equilibrium and on ductile mechanisms developing under monotonically imposed displacements. The conclusions of this study need to be verified by analyses of the dynamic response of systems so designed and subjected to representative earthquake records. A number of studies with this specific aim addressing particularly the quantification of the effect of the rotary inertia of distributed mass are currently being undertaken. Results are expected in the near future.

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