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Development of Bridge Fragility Curves Based on Damage Data

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ABSTRACT

This paper presents methods of bridge fragility curve development on the basis of damage data obtained from the past earthquakes, particularly the 1994 Northridge and the 1995 Hyogo-ken Nanbu (Kobe) earthquake. These fragility curves are extremely useful for calibrating the fragility information that must be developed primarily on an analytical basis for similar bridges. Two-parameter lognormal distribution functions are used to represent the fragility curves. These two parameters (referred to as fragility parameters) are estimated by two distinct methods. The first method is more traditional and uses the maximum likelihood procedure treating each event of bridge damage as a realization from a Bernoulli experiment. The second method is unique in that it permits simultaneous estimation of the fragility parameters of the family of fragility curves, each representing a particular state of damage, associated with a population of bridges. The second method still utilizes the maximum likelihood procedure. In this case, however, each event of bridge damage is treated as a realization from a multi-outcome Bernoulli type experiment. These two methods of parameter estimation are used for each of the populations of bridges inspected for damage after the Northridge and the Kobe earthquake.

INTRODUCTION

Bridges are potentially one of the most seismically vulnerable structures in the highway system. While performing a seismic risk analysis of a highway system, therefore, it is imperative to identify seismic vulnerability of bridges associated with various states of damage. The development of vulnerability information in the form of fragility curves is a widely practiced approach when the information is to be developed accounting for a multitude of uncertain sources involved, for example, in estimation of seismic hazard, structural characteristics, soil-structure interaction, and site conditions.

The major effort of this study is placed on the development of empirical fragility curves by utilizing the damage data associated with past earthquakes. At the same time, it introduces statistical procedures appropriate for the development of fragility curves under the assumption that they can be represented by two-parameter lognormal distribution functions with the unknown median and log-standard deviation. These two parameters are referred to as the fragility parameters in this study. Two different sets of procedures describe how the fragility parameters are estimated. The one procedure (Method 1) is used when the fragility curves are independently developed for different states of damage, while the other (Method 2) when they are constructed dependently each other in such a way that the log-standard deviation is common to all the fragility curves. These fragility curves are developed utilizing bridge damage data obtained from the past earthquakes, specifically the 1994 Northridge and the 1995 Hyogo-ken Nanbu (Kobe) earthquake.

Two-parameter lognormal distribution functions were traditionally used for fragility curve construction. This was motivated by its mathematical expedience in approximately relating the actual structural strength capacity with the design strength through an overall factor of safety which can be assumedly factored into a number of multiplicative safety factors, each associated with a specific source of uncertainty. When the lognormal assumption is made for each of these factors, the overall safety factor also distributes lognormally due to the multiplicative reproducibility of the lognormal variables. This indeed was the underpinning assumption that was made in the development of probabilistic risk assessment methodology for nuclear power plants in the 1970's and in the early 1980's [1]. Although this assumption is not explicitly used in this report, fragility curves are also modeled by lognormal distribution function in this study. Use of the three-parameter lognormal distribution functions for fragility curves is possible with the third parameter estimating the threshold of ground motion intensity below which the structure will never sustain any damage. However, this has never been a popular option primarily because no one wishes to make such a definitive and potentially unconservative assumption. In passing, it is mentioned that Shinozuka, *et al.* [2] carried out additional studies where the methods are developed to assess the goodness of fit of the fragility curves to the data and to estimate confidence intervals for the fragility parameters, and some preliminary evaluations are made on the significance of the fragility curves developed as a function of ground intensity measures other than PGA.

The reader is also referred to a list of references in Shinozuka, *et al.* [2] for the previous analyses performed by different authors on fragility curves developed for civil structures with different emphases.

EMPIRICAL FRAGILITY CURVES

It is assumed that the empirical fragility curves can be expressed in the form of two-parameter lognormal distribution functions, and developed as functions of peak ground acceleration (PGA) representing the intensity of the seismic ground motion. Use of PGA for this purpose is considered reasonable since it is not feasible to evaluate spectral acceleration by identifying significantly participating natural modes of vibration for each of the large number of bridges considered for the analysis here, without having a corresponding reliable ground motion time history. The PGA value at each bridge location is determined by interpolation and extrapolation from the PGA data due to D. Wald of USGS [3].

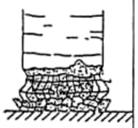
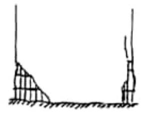
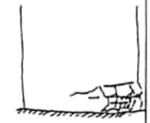
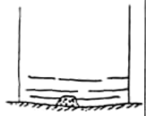
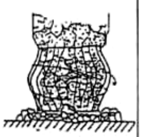
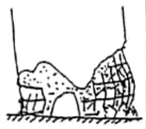
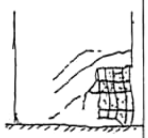
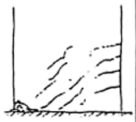
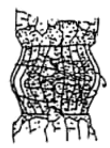
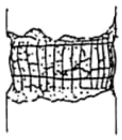
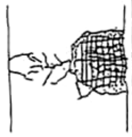
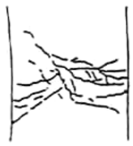
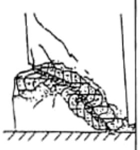
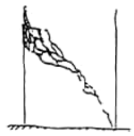
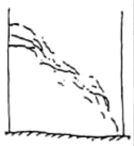
For the development of empirical fragility curves, the damage inspection data are usually utilized to establish the relationship between the ground motion intensity and the damage state of each bridge. This is also the case for the present study. One typical page of such damage data for the Caltrans' bridges under the Northridge event is shown in Table 1, where the extent of damage is classified in column 5 into the state of no, minor, moderate and major damage in addition to the state of collapse. This database did not provide explicit physical definitions of these damage states (in column 5, a blank space signifies no damage). As far as the Caltrans' bridges are concerned, this inspection database is used when a damage state is assigned to each bridge in the analysis that follows. This inspection database was developed on the basis of Caltran's damage reports [4,5] with PGA data provided by Wald (1998). In view of the time constraint in which the inspection had to be completed after the earthquake, the classification of each bridge into one of the five damage states, understandably, contains some elements of judgement.

Hanshin Expressway Public Corporation's (HEPC's) report on the damage sustained by RC bridge columns resulting from the Kobe earthquake uses five classes of damage state as shown in Fig. 1 in which the damage states As, A, B, C and D are defined by the corresponding sketches of damage within each of four failure modes. It appears reasonable to consider that these damage states respectively represent states of collapse (As), major damage (A), moderate damage (B), minor damage (C) and no damage (D).

In this study, the fragility parameter estimation is carried out in two different ways. The first method (Method 1) independently develops a fragility curve for each damage state for each sample of bridges with a given set of bridge attributes. A family of four fragility curves can, for example, be developed independently for the damage states respectively identified as "at least minor",

Table 1 Northridge earthquake damage data

BRIDGE _NO	YEAR BUILT	LENGTH (ft)	DECK_WD (ft)	DAMAGE STATE	PGA (g) D. Wald	SOIL TYPE	NO. OF SPANS	SKEW (DEG.)	HINGE JOINT	BENT JOINT
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
53 1783	1967	318	547	MAJ	0.61	C	2	40	0	0
53 1784	1967	156	1670		0.09	C	4	4	0	0
53 1785	1967	155	1480		0.09	C	3	7	0	0
53 1786	1967	155	1680		0.11	C	3	4	0	0
53 1789	1967	219	1207		0.10	C	2	5	0	0
53 1790	1967	1511	1380	MIN	0.29	C	14	9	4	0
53 1790H	1967	2831	280	MOD	0.29	C	27	99	13	0
53 1792L	1967	146	680	MAJ	0.64	C	1	32	0	0
53 1792R	1967	146	680	MIN	0.64	C	1	32	0	0
53 1796	1967	220	395	MOD	0.68	C	2	0	0	0
53 1797L	1967	741	68	COL	0.68	C	5	67	2	0
53 1797R	1967	741	68	COL	0.68	C	5	67	2	0

Damage State / Damage Mode	As	A	B	C	D	Remarks
1. Bending Damage at ground level	Damage through entire cross section 	Damage mainly at two opposite sides 	Damage mainly at one side 	Light cracking and Partial Spalling 	No Damage	This mode unltimately produces buckling of rebars, spalling and crushing of core concrete
2. Combined bending & sear Damage at ground level	Internal Damage 	Damage at two sides 	Damage mainly at one side 	Light cracking and Partial Spalling 	No damage	Bending and shear cracks progress with more wide-spread spalling than model and hoops detached from anchorage
3. Combined bending & shear Damage at the level of reduction of longitudinal rebars	Internal Damage 	Internal damage 	Damage mainly at one side 	Partial damage 	No damage	Damage and collapse are observed at about the location (typically 4-5m above ground) of reduction of longitudinal rebars, accompanying buckling of rebars and detached hoops.
4. Shear Damage at ground level	Damage through entire cross-section 	Damage through column 	Partial Damage 	Light cracking* 	No Damage	Columns with low aspect ration sheared at 45° angle

* No description provided in the original

Fig. 1 Description of states of damage for Hanshin Expressway Corporation's bridge columns "at least moderate", "at least major" and "collapse", making use of the entire sample (of size equal to 1,998) of Caltrans' expressway bridges in Los Angeles County, California subjected to the Northridge earthquake and inspected for damage after the earthquake. This is done by estimating, by the maximum likelihood method, the two fragility parameters of each lognormal distribution function representing a fragility curve for a specific state of damage. These fragility curves are valid under the assumption that the entire sample is statistically homogeneous. The same independent estimation procedure can be applied to samples of bridges more realistically categorized. A sample consisting only of single span bridges out of the entire sample is such a case for which four fragility curves can also be independently developed for all the bridges with a single span. Method 1 also includes in Shinozuka, *et al.* [2] the procedure to test the hypothesis that the observed damage data are generated by chance from the corresponding fragility curves thus developed (test of goodness of fit). In addition, Method 1 provides in Shinozuka, *et al.* [2] a procedure of estimating statistical confidence intervals of the fragility parameters through a Monte Carlo simulation technique.

It is noted that the bridges in a state of damage as defined above include a sub-set of the bridges in a severer state of damage implying that the fragility curves developed for different states of damage within a sample are not supposed to intersect. Intersection of fragility curves can happen, however, under the assumption that they are all represented by lognormal distribution functions and constructed independently, unless log-standard derivations are identical for all the fragility curves. This observation leads to the following method referred to as Method 2, where the parameters of the lognormal distribution functions representing different states of damage are simultaneously estimated by means of the maximum likelihood method. In this method, the parameters to be estimated are the median of each fragility curve and one value of the log-standard derivation prescribed to be common to all the fragility curves. The procedures of hypothesis testing and confidence interval estimation associated with Method 2 are also given in Shinozuka, *et al.* [2].

PARAMETER ESTIMATION; METHOD 1

In Method 1, the parameters of each fragility curve are independently estimated by means of the maximum likelihood procedure as described below. The likelihood function for the present purpose is expressed as

$$L = \prod_{i=1}^{N} [F(a_i)]^{x_i} [1 - F(a_i)]^{1-x_i} \quad (1)$$

where $F(\cdot)$ represents the fragility curve for a specific state of damage, a_i is the PGA value to which bridge i is subjected, x_i represents realizations of the Bernoulli random variable X_i and $x_i = 1$ or 0 depending on whether or not the bridge sustains the state of damage under PGA = a_i, and N is the total number of bridges inspected after the earthquake. Under the current lognormal assumption, $F(a)$ takes the following analytical form

$$F(a) = \Phi\left[\frac{\ln\left(\frac{a}{c}\right)}{\zeta}\right] \tag{2}$$

in which "a" represents PGA and $\Phi[\cdot]$ is the standardized normal distribution function. The two parameters c and ζ in Eq. (2) are computed as c_0 and ζ_0 satisfying the following equations to maximize $\ln L$ and hence L;

$$\frac{d\ln L}{dc} = \frac{d\ln L}{d\zeta} = 0 \tag{3}$$

This computation is performed by implementing a straightforward optimization algorithm.

PARAMETER ESTIMATION; METHOD 2

A set of parameters of lognormal distributions representing fragility curves associated with all levels of damage state involved in the sample of bridges under consideration are estimated simultaneously in Method 2. A common log-standard deviation is estimated along with the medians of the lognormal distributions with the aid of the maximum likelihood method. The common log-standard deviation forces the fragility curves not to intersect. The following likelihood formulation is developed for the purpose of Method 2.

Although Method 2 can be used for any number of damage states, it is assumed here for the ease of demonstration of analytical procedure that there are four states of damage including the state of no damage. A family of three (3) fragility curves exist in this case as schematically shown in Fig. 2 where events E_1, E_2, E_3 and E_4 respectively indicate the state of no, minor, moderate and major damage. $P_{ik} = P(a_i, E_k)$ in turn indicates the probability that a bridge i selected randomly from the sample will be in the damage state E_k when subjected to ground motion intensity expressed by PGA $= a_i$. All fragility curves are represented by two-parameter lognormal distribution functions

$$F_j(a_i; c_j, \varsigma_j) = \Phi\left[\frac{\ln(a_i / c_j)}{\varsigma_j}\right] \tag{4}$$

where c_j and ζ_j are the median and log-standard deviation of the fragility curves for the damage state of "at least minor", "at least moderate" and "major" identified by j = 1, 2 and 3 respectively. From this definition of fragility curves, and under the assumption that the log-standard deviation is equal to ζ common to all the fragility curves, one obtains;

$$P_{i1} = P(a_i, E_1) = 1 - F_1(a_i; c_1, \zeta) \tag{5}$$

$$P_{i2} = P(a_i, E_2) = F_1(a_i; c_1, \zeta) - F_2(a_i; c_2, \zeta) \tag{6}$$

$$P_{i3} = P(a_i, E_3) = F_2(a_i; c_2, \zeta) - F_2(a_i; c_3, \zeta) \tag{7}$$

$$P_{i4} = P(a_i, E_4) = F_3(a_i; c_3, \zeta) \tag{8}$$

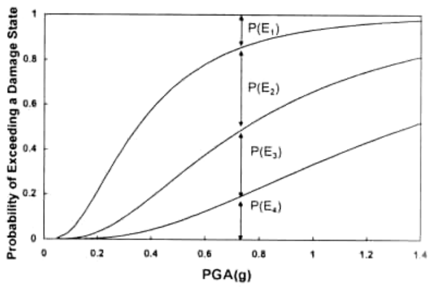


Fig. 2 Schematics of fragility curves

The likelihood function can then be introduced as

$$L(c_1, c_2, c_3, \zeta) = \prod_{i=1}^{n} \prod_{k=1}^{4} P_k(a_i; E_k)^{x_{ik}} \tag{9}$$

where

$$x_{ik} = 1 \quad \text{or} \quad x_{ik} = 0 \tag{10}$$

depending on whether or not the damage state E_k occurs for the i-th bridge subjected to $a = a_i$. The maximum likelihood estimates c_{0j} for c_j and ζ_0 for ζ are obtained by solving the following equations,

$$\frac{\partial \ln L(c_1, c_2, c_3, \zeta)}{\partial c_j} = \frac{\partial \ln L(c_1, c_2, c_3, \zeta)}{\partial \zeta} = 0 \qquad (j = 1, 2, 3) \tag{11}$$

by again implementing a straightforward optimization algorithm.

FRAGILITY CURVES FOR CALTRANS' AND HEPC'S BRIDGES

Four fragility curves for Caltrans' bridges associated with the four states of damages are plotted in Figs. 3 and 4, upon estimating the parameters involved by Methods 1 and 2 respectively (with their respective median and log-standard deviation values also indicated). As mentioned earlier, a sample of 1,998 bridges are used to develop these fragility curves.

Fragility curves are also constructed [6] and shown in Fig. 5 (Method 1) and Fig. 6 (Method 2) on the basis of a sample of 770 single-support reinforced concrete columns along two stretches of the viaduct, one in the HEPC's Kobe Route and the other in the Ikeda Route with total length of approximately 40km. A similar inspection database to Table 1 (see Shinozuka, *et al.*, [2] for the database) was used for this purpose. These bridge columns are of similar geometry and similarly reinforced. In this respect, the 770 columns under consideration here constitute a much more homogeneous statistical sample than the Caltrans' bridges considered earlier. The PGA value at each column location under the Kobe earthquake is estimated by Nakamura, *et al.* [6] on the basis of the work by Nakamura, *et al.* [7].

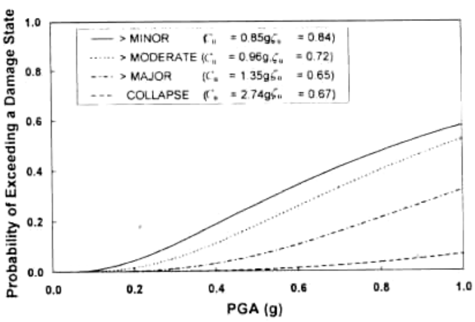


Fig. 3 Fragility curves for Caltrans' bridges (Method 1)

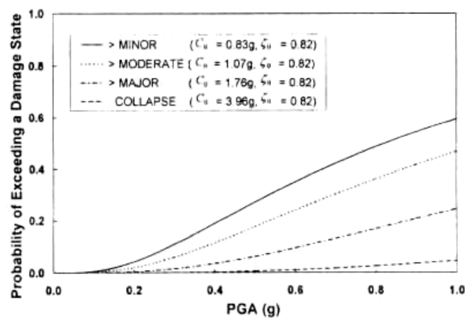


Fig. 4 Fragility curves for Caltrans' bridges (Method 2)

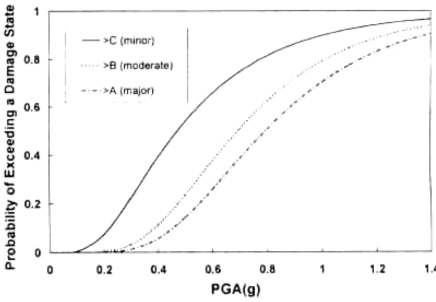


Fig. 5 Fragility curves for HEPC's bridge columns (Method 1)

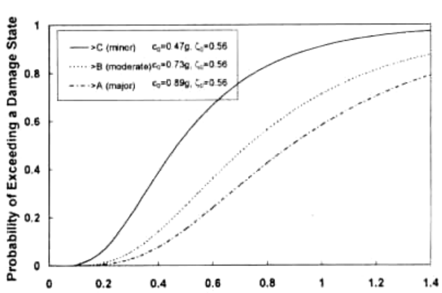


Fig. 6 Fragility curves for HEPC's bridge columns (Method 2)

FRAGILITY CURVES FOR STRUCTURAL SUB-SETS OF CALTRANS' BRIDGES

In the preceding analysis, it was assumed that the sample of bridges inspected after the earthquake is statistically homogeneous. This assumption is not quite reasonable for the Caltrans' bridges, while it is reasonable for the HEPC's bridge columns as mentioned earlier. In the present study, therefore, the sample of the HEPC's bridge columns considered is treated statistically as homogeneous and Figs. 5 and 6 represent the families of fragility curves assignable to any bridge column arbitrarily chosen from the underlying homogenous population of bridge columns. For the mathematical reasons mentioned earlier, it is recommended even then that the fragility curves (Fig. 6) obtained by means of Method 2 be considered for applications, although a statistical analysis indicates that the fragility curves (Fig. 5) obtained by Method 1 cannot mathematically be rejected [2]. As opposed to the case of HEPC's bridge columns, the statistical homogeneity would be an oversimplification for the sample of the Caltrans' bridges. In fact, it is reasonable to sub-divide the sample of the Caltrans' bridges into a number of sub-sets in accordance with the pertinent bridge attributes and their combinations. This should be done in such a way that each sub-sample can be considered to be drawn from the corresponding sub-population which is more homogeneous than the initial population. In this regard, it is recognized each bridge can easily be associated with one of the following three distinct attributes; (A) It is either single span (S) or multiple span (M) bridge, (B) it is built on either hard soil (S_1), medium soil (S_2) or soft soil (S_3) in the definition of UBC 93, and (C) it has a skew angle θ_1 (less than 20°), θ_2 (between 20° and 60°) or θ_3 (larger than 60°). The sample can then be sub-divided into a number of sub-sets. To begin with, one might consider the first level hypothesis that the entire sample is taken from a statistically homogenous population of bridges. The second level sub-sets are created by dividing the sample either (A) into two groups of bridges, one with single spans and the other with multiple spans, (B) into three groups, the first with soil condition S_1, the second with S_2 and the third with S_3, or (C) into three groups depending on the skew angles θ_1, θ_2 and θ_3. The third level sub-sets consists of either (D) 6 groups each with a particular combination between (S, M) and (S_1, S_2, S_3), (E) 6 groups each with a combination between (S, M) and (θ_1, θ_2, θ_3), or (F) 9 groups each with a combination between (θ_1, θ_2, θ_3) and (S_1, S_2, S_3). Finally, the fourth level sub-sets comprises of 18 groups each with a combination of the attributes (S, M), (S_1, S_2, S_3) and (θ_1, θ_2, θ_3).

The first level represents nothing but the entire sample taken from the underlying homogeneous population. The fragility curves are developed under this assumption in Figs. 3 and 4 for the Caltrans' bridges. The second, third and fourth level sub-sets are all considered and analyzed for the fragility curve development with the aid of Method 2 in Shinzuka *et al.*, [2]. The median values and log-standard deviations of the first two levels of attribute combinations are listed in Table 2. Note that, if an element of a matrix in Table 2 shows NA, it indicates that null sub-sample was found for the particular combination of bridge attributes the element signifies. The families of fragility curves corresponding to the second level subsets with skewness as the attribute (Table 2c) are plotted in Figs. 7 ~ 9. They show that, the larger the skew angle, the more fragile bridges are. These fragility curves classified in accordance with pertinent structural and geotechnical attributes play a pivotal role in the seismic performance assessment of the expressway network [2].

Table 2 Median and log-standard deviation at different levels of sample sub-division

(a) First Level (Composite)

Median	Log. St. Dev.
0.83	0.82
1.07	0.82
1.76	0.82
3.96	0.82

(b) Second Level (Span)

	Median	Log. St. Dev.
Sigle	1.22	0.78
	1.60	0.78
	2.65	0.78
	10.00	0.78
Multiple	0.72	0.78
	0.92	0.78
	1.51	0.78
	3.26	0.78

(c) Second Level (Skew)

	Median	Log. St. Dev.
Sk1 0° ~ 20°	0.99	0.95
	1.38	0.95
	2.52	0.95
	5.15	0.95
Sk2 20° ~ 60°	0.71	0.73
	0.87	0.73
	1.38	0.73
	3.93	0.73
Sk3 > 60°	0.50	0.59
	0.63	0.59
	0.93	0.59
	1.69	0.59

(d) Second Level (Soil)

	Median	Log. St. Dev.
Soil A	1.35	0.94
	1.79	0.94
	2.62	0.94
	10.00	0.94
Soil B	0.97	0.94
	1.36	0.94
	2.19	0.94
	10.00	0.94
Soil C	0.79	0.79
	1.01	0.79
	1.70	0.79
	3.57	0.79

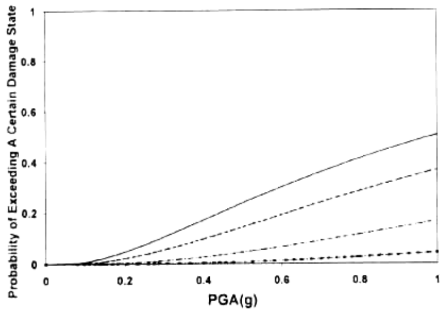


Fig. 7 Fragility curves for a second level subset (Caltrans' bridges; skew > 60°) by Method 2

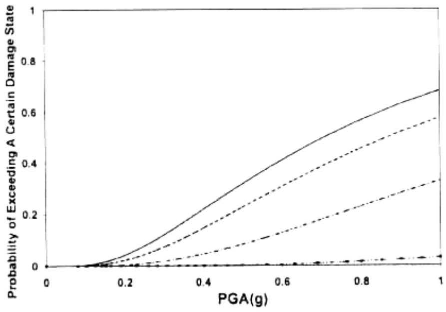


Fig. 8 Fragility curves for a second level subset (Caltrans' bridges; 20° < skew < 60°) by Method 2

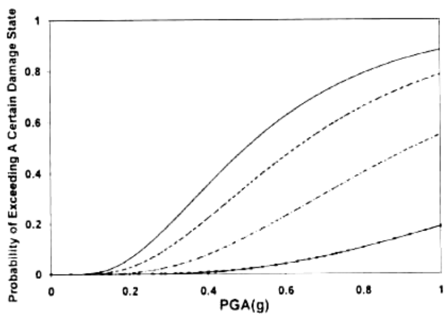


Fig. 9 Fragility curves for a second level subset (Caltrans' bridges; skew ≤ 20°) by Method 2

CONCLUSIONS

This paper presents methods of bridge fragility curve development utilizing bridge damage data obtained from the past earthquakes, particularly the 1994 Northridge and the 1995 Hyogo-ken Nanbu (Kobe) earthquake. Two-parameter lognormal distribution functions are used to represent the fragility curves. These two parameters (referred to as fragility parameters) are estimated by two distinct methods. The first method is more traditional and uses the maximum likelihood procedure treating each event of bridge damage as a realization from a Bernoulli experiment, while the second method is unique in that it permits simultaneous estimation of the fragility parameters of the family of fragility curves, each representing a particular state of damage, associated with a population of bridges. The second method still utilizes the maximum likelihood procedure, however, with each event of bridge damage treated as a realization from a multi-outcome Bernoulli type experiment. These two methods of parameter estimation are used for each of the populations of bridges inspected for damage after the Northridge and the Kobe earthquake. This paper also studied the fragility curves of the bridges classified according to some relevant structural and geotechnical attributes. While the author is hopeful that the conceptual and theoretical treatment dealt in this study can provide theoretical basis and analytical tools of practical usefulness for the development of empirical fragility curves, there are many analytical and implementational aspects that require further study including;

1. Physical definition of damage that can be used for post-earthquake damage inspection and analysis.

2. Use of other measures of ground motion intensity than PGA for fragility curve development.
3. Bridge categorization based on physical attributes.
4. Further study on the use of nonlinear static analysis procedures for fragility curve development.
5. Transportation systems analysis accounting for uncertainty in the fragility parameters.

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